Geometry & Gravity Space related 3D Integrated Geomonitoring - Feasibility, Advantages and Implementation into the GOCA-System -

Prof. Dr.-Ing. Reiner Jäger

Hochschule Karlsruhe Technik und Wirtschaft (HSKA) - University o Applied Sciences
Faculty for Informationsmanagement and Media (IMM)
Dean of Master Study Programmes Geomatik&Geomatics (MSc)

Head of Institute of Geomatics and Projectleader at Institute for Applied Research (IAF)

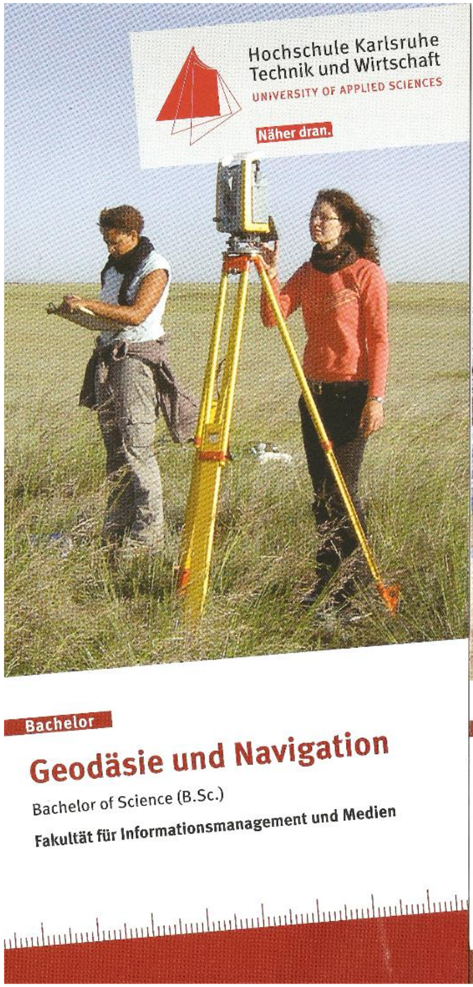
RaD

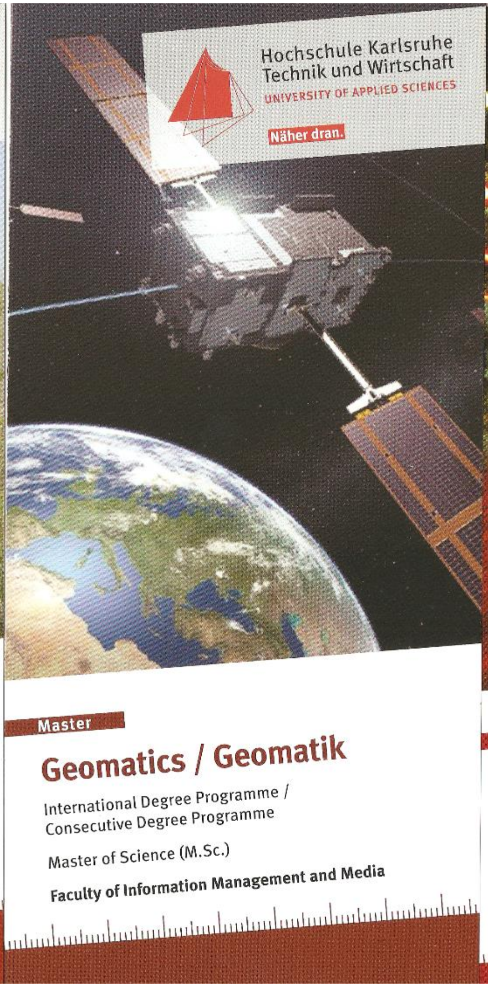
www.dfhbf.de , www.goca.info , www.monika.ag, www.geozilla.de, www.moldpos.eu
www.navka.de

Honorary Professor Siberian State University of Geosystems and Technologies

Email: reiner.jaeger@hs-karlsruhe.de

Study Programmes of the Faculty IMM with Contents related to Geodesy and Navigation (www.hs-karlsruhe.de) Research Oriented (BSc – MSc – PhD)





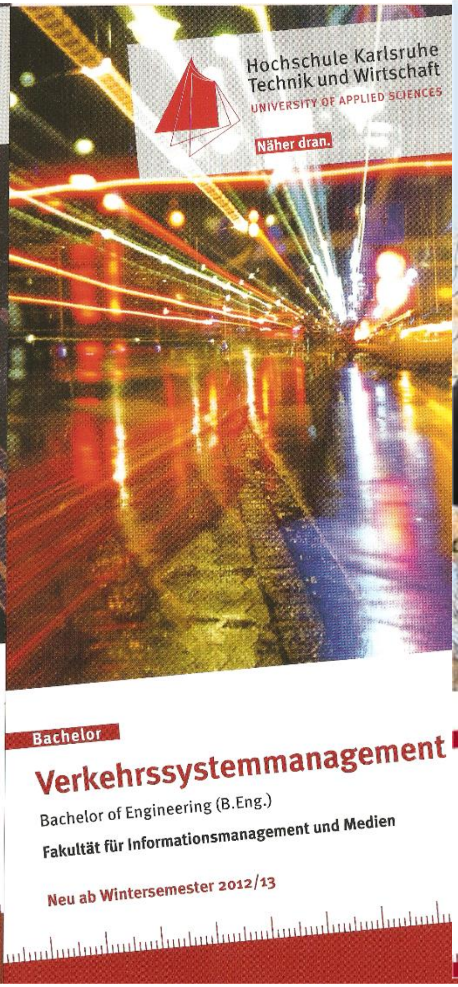
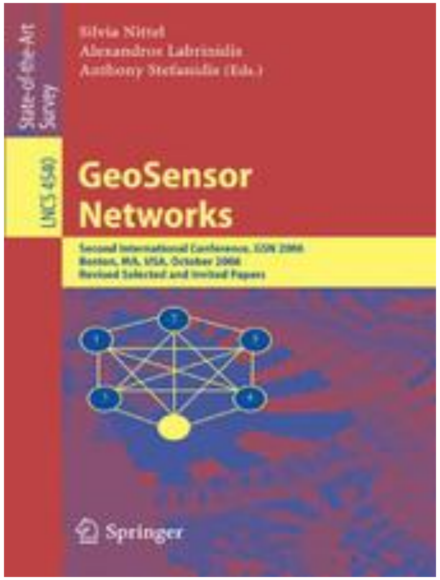


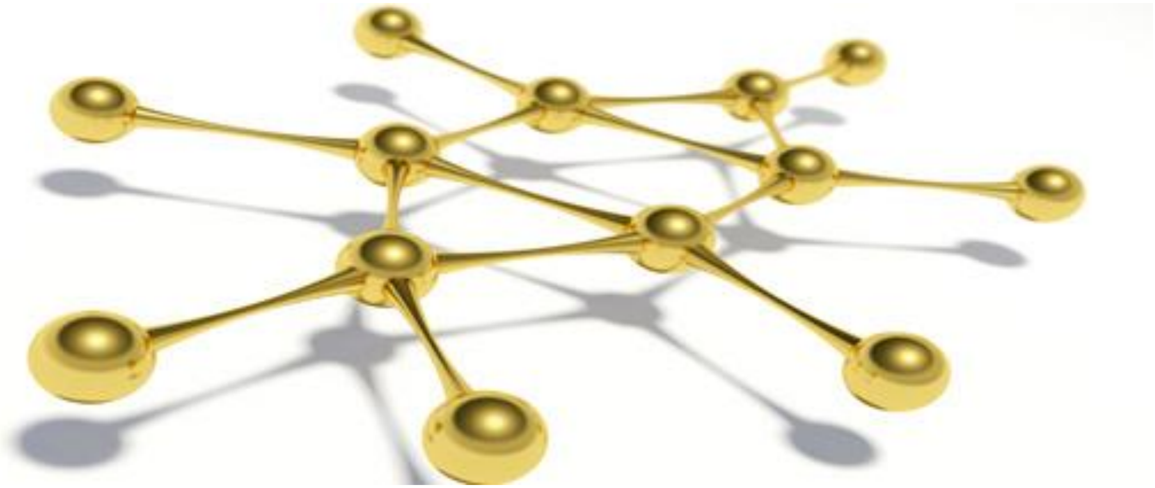
FIG
Modeling Standards

on

Geodetic Monitoring

Geosensornetworks and Geomonitoring





Active Nodes/Sensors

- Collect data actively themselves and send it further through the network

Passive Nodes/Sensors

- Collect and send data based on an external sensor control

Standards an Developments in Geodetic Monitoring

Mathematical Model: Relation between the Observation Data (l) und Object State Parameters y. Stochastical Models C_l of the Observation Errors ε at two general observation times (epochs) t_1 and t_2

$$\mathbf{l}(t_1) - \varepsilon(t_1) = \tilde{\mathbf{l}}(\mathbf{y}(t_1)) \quad ; \quad \mathbf{C}_l(t_1)$$
$$\mathbf{l}(t_2) - \varepsilon(t_2) = \tilde{\mathbf{l}}(\mathbf{y}(t_2)) \quad ; \quad \mathbf{C}_l(t_2)$$

Parameter- Estimation (after linearisation with approximative parameters y^0)

Approach: $$\sum_{i=1}^{n} \rho(\bar{v}_i) = \sum_{i=1}^{n} \rho\left((\mathbf{C}_l^{-\frac{1}{2}} \cdot \mathbf{A})_i \cdot d\hat{\mathbf{y}} - (\mathbf{C}_l^{-\frac{1}{2}} \cdot (\mathbf{l} - \mathbf{l}(\mathbf{y}^0)))_i \right) = \mathrm{Min}|_{d\hat{y}}$$

Choice of the Estimation Principle: $$\boxed{\rho(\bar{v}_i) = \frac{1}{2}\bar{v}_i^2} \quad \left(\boxed{\rho(\bar{v}_i) = \frac{1}{2}|\bar{v}_i|} \ \ldots \ \boxed{\rho(\bar{v}_i) = \begin{cases} \frac{1}{2}\bar{v}_i^2 & \forall\ |\bar{v}_i| \le k \\ |\bar{v}_i| & \forall\ |\bar{v}_i| > k \end{cases}} \right)$$

Result = State vector of Parameters y(t) $$\boxed{\hat{y} = y^0 + d\hat{y}}$$

Absolute Geodetic Deformation Network

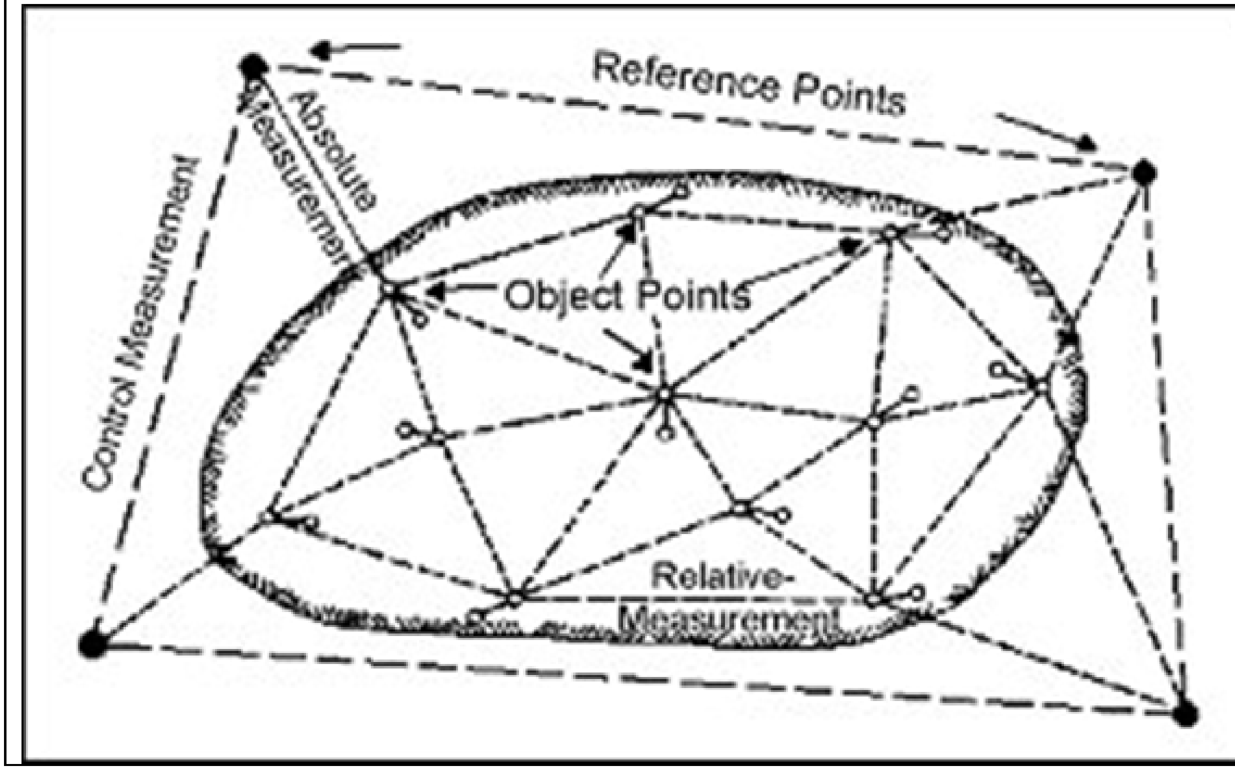


Figure and definition of an absolute deformation network according to Pelzer (1972)

Positions $x(t)$ and **displacements $u(t)$** of the so-called object-points $\mathbf{x}_0$ and their covariance matrices $\mathbf{C}_{x_0}$ and $\mathbf{C}_{u_0}$ are determined and geo-referenced in a connectedly set-up and unique coordinate frame set up by the reference points x_R.

The reference frame x_R is generally a free network (undistorted by the influence of classical fix-points). The defect "d" of the free monitoring network, is depending on the relative observation types. Exception: ITRF-frames are set-up by VLBI and GNSS and are inertial space based "absolute" networks (defect d=0).

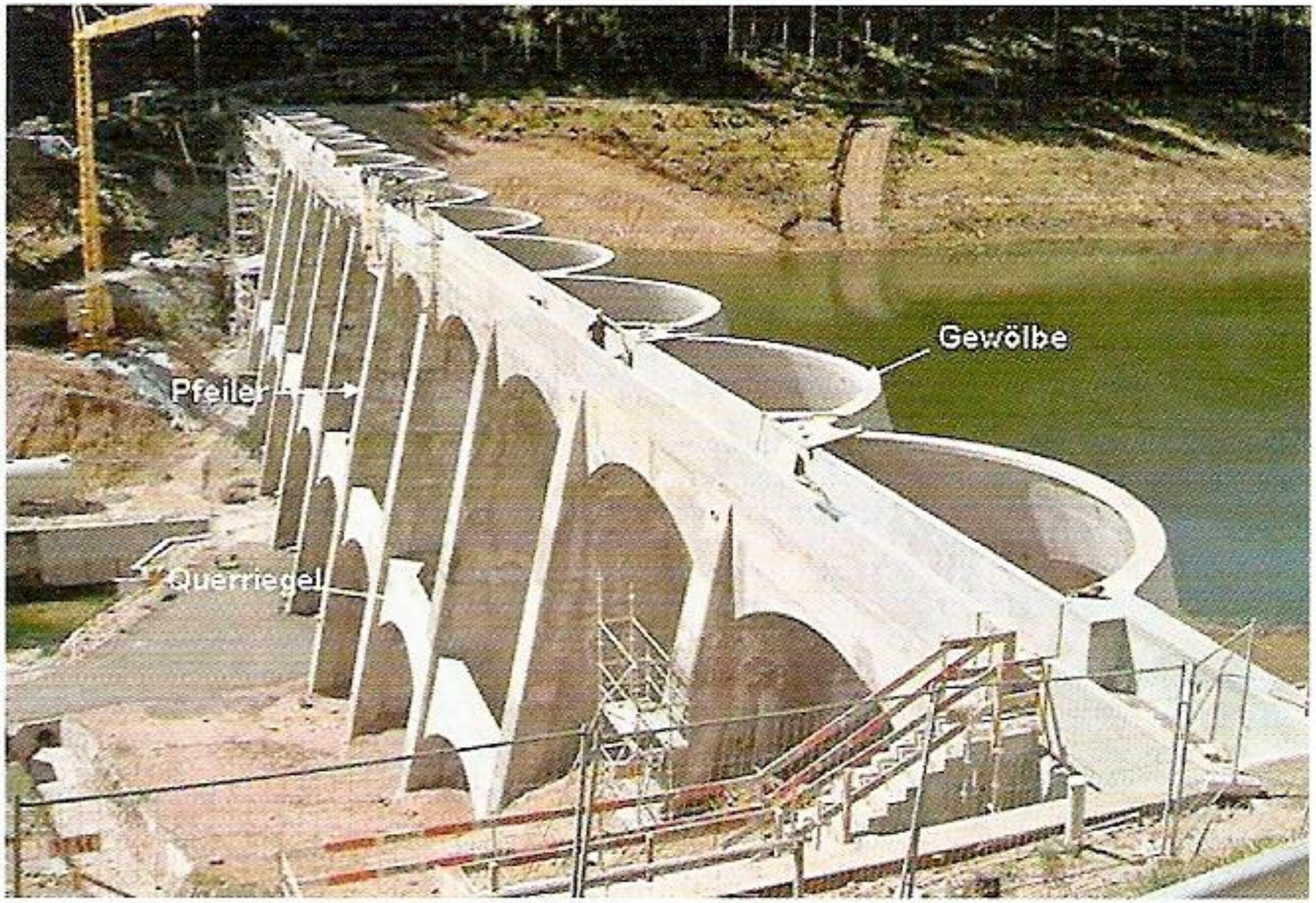
The object-point positions $\mathbf{x}_0(t)$ and their covariance matrix $\mathbf{C}_{x_0}$, as well as the displacements $\mathbf{u}(t)$ can however always be transformed to another frame by a similarity transformation, e.g. to a building system.

Relative Geodetic Monitoring Deformation Network

Typically for GNSS-reference stations network of GNSS-positioning services

Characteristics and Definition: Congruency the reference points x_R as basic hypothesis for a geodetic monitoring. Testing of reference point displacements $\nabla x_R(t) = u_R(t)$. Possible for free and non-defect absolute networks (ITRF).

Observation based geodetic monitoring

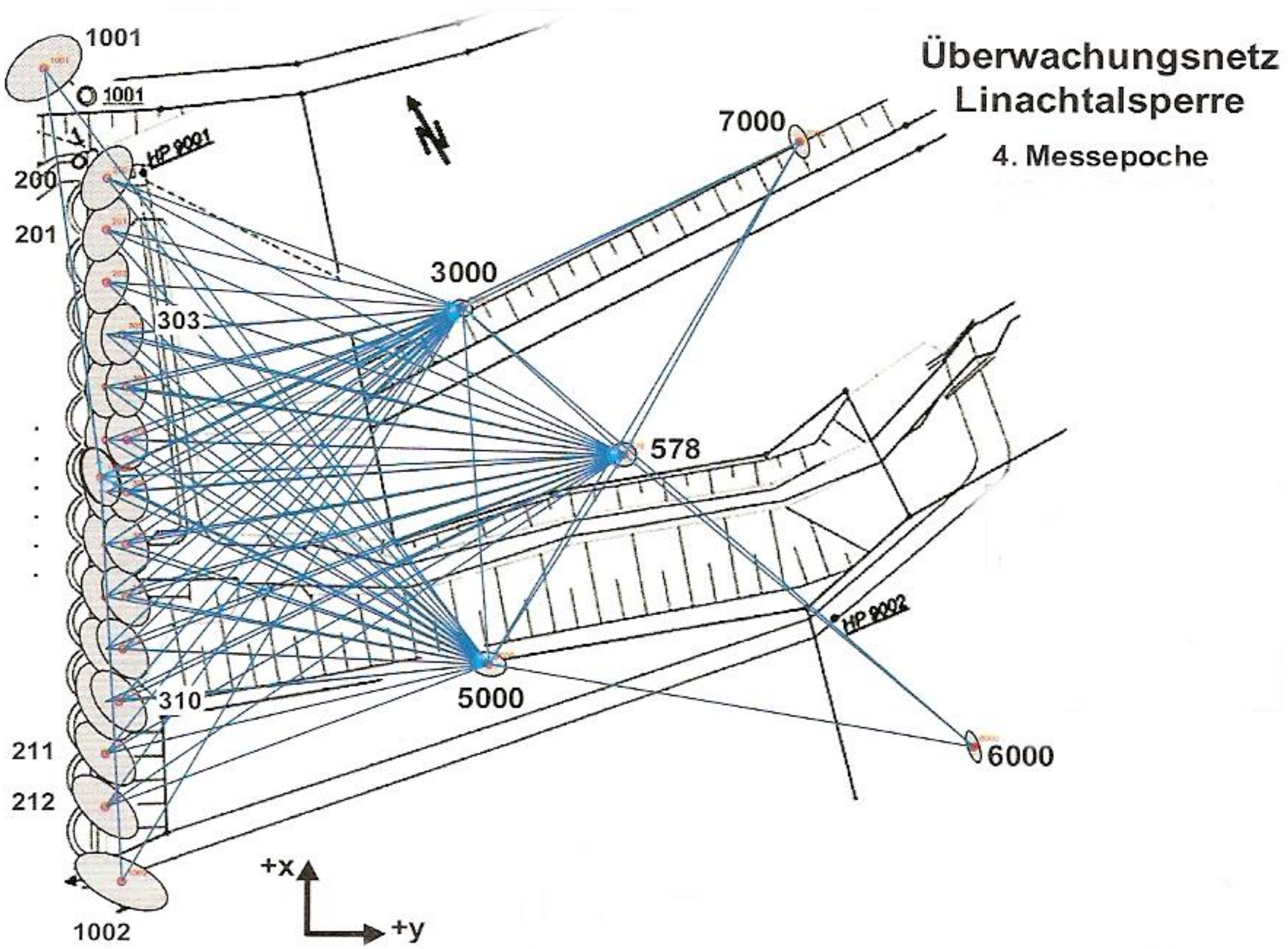


Linachtalsperre

Standards of Geodetic Monitoring (Deformationsanalysis)
FIG Commission 6 and Working Groups (~ 1975 - 2009)

Absolute Deformation Network : Partitioning into Stable Points x_R und Object Points x_O

Observation based geodetic monitoring (GOCA)



GOCA Software / System
www.goca.info

Standards of Geodetic Monitoring (Deformationsanalysis)
FIG Commission 6 and Working Groups (~ 1975 - 2009)

Network Adjustment with all Measurements in epochs or intervals. Coordinates in the stable area x_R identical in all epochs, object-point coordinates x_O (t) time-dependent and changing

Observation based geodetic monitoring (GOCA)

$y = (x_R;\, x_{0,1;} x_{02})$ – Coordinates x(t)

$$(\mathbf{l}_1 - \mathbf{l}_1(\mathbf{y}^0)) + \mathbf{v}_1 = \mathbf{A}_{R,1} \cdot d\mathbf{x}_R + \mathbf{A}_{O,1} \cdot d\mathbf{x}_{O,1} + \mathbf{0} \cdot d\mathbf{x}_{O,2}$$

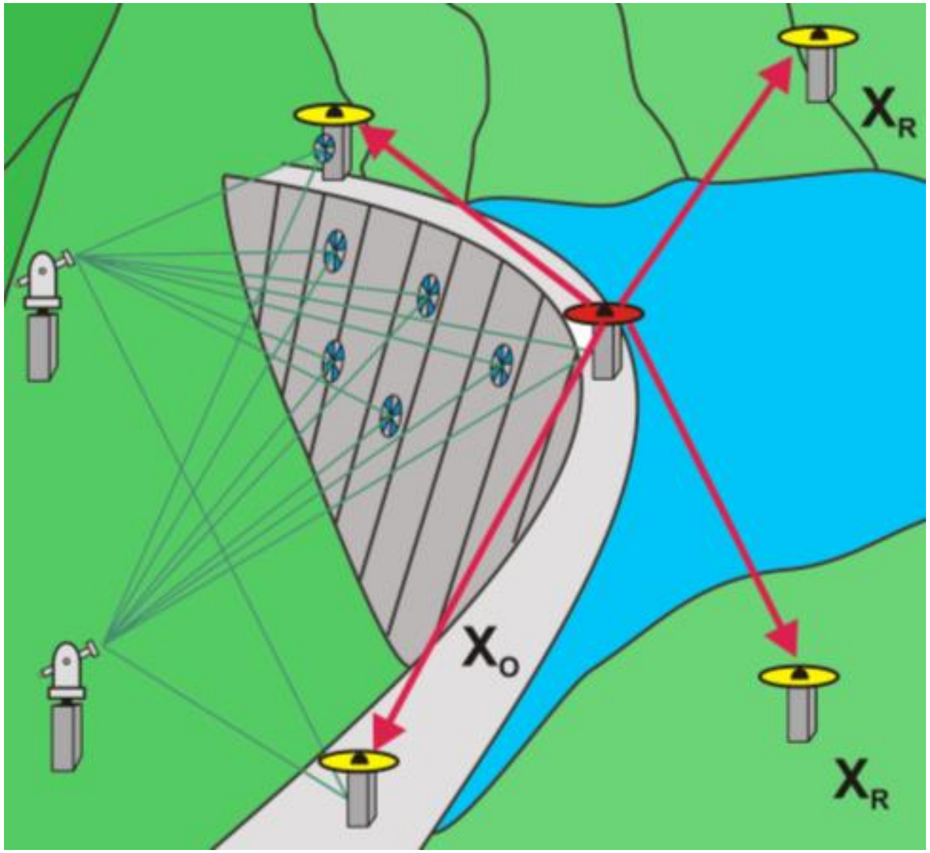
$$(\mathbf{l}_2 - \mathbf{l}_2(\mathbf{y}^0)) + \mathbf{v}_2 = \mathbf{A}_{R,2} \cdot d\mathbf{x}_R + \mathbf{0} \cdot d\mathbf{x}_{O,1} + \mathbf{A}_{O,2} \cdot d\mathbf{x}_{O,2}$$

Least Squares Adjustment

$$d\hat{\mathbf{y}} = (\mathbf{A}^T \mathbf{C}_l^{-1} \mathbf{A})^{-} \mathbf{A}^T \mathbf{C}_l^{-1} \cdot (\mathbf{l} - \mathbf{l}(\mathbf{y}^0))$$

$$\hat{\mathbf{y}}(t_1, t_2) = y^0 + d\hat{\mathbf{y}} = \begin{bmatrix} \mathbf{x}_R^0 + d\mathbf{x}_R \\ \mathbf{x}_{O1}^0 + d\mathbf{x}_{O1} \\ \mathbf{x}_{O2}^0 + d\mathbf{x}_{O2} \end{bmatrix}$$

$$Cy = \left[\begin{array}{c|cc} \mathbf{A}_R^T \mathbf{C}_l^{-1} \mathbf{A}_R & \mathbf{A}_R^T \mathbf{C}_l^{-1} \mathbf{A}_{O1} & \mathbf{A}_R^T \mathbf{C}_l^{-1} \mathbf{A}_{O2} \\ \hline \mathbf{A}_{O1}^T \mathbf{C}_l^{-1} \mathbf{A}_R & \mathbf{A}_{O1}^T \mathbf{C}_l^{-1} \mathbf{A}_{O1} & 0 \\ \mathbf{A}_{O2}^T \mathbf{C}_l^{-1} \mathbf{A}_R & \mathbf{0} & \mathbf{A}_{O2}^T \mathbf{C}_l^{-1} \mathbf{A}_{O2} \end{array}\right]^{-}$$



Discrete State Parameters y

$x_o(t)$ - Object point coordinates

$u_o(t,x_i)$ - Object point displacements

Coordinate-related geodetic monitoring

Epoch 1: $\mathbf{x}(\mathbf{l}_1)$ and $\mathbf{C}_{\mathbf{x}1}$ from epoch adjutment

Epoch 2: $\mathbf{x}(\mathbf{l}_2)$ and $\mathbf{C}_{\mathbf{x}2}$ from epoch adjustment

Condition for equivalence with observation related approach in case of free networks:

In case of different approximate coordinates x_0 a defect (d) dependent „Helmert-Transformation on new defined approximate coordinates x_0 has to be done together with an S-Transformation related to x_O

$$\mathbf{dx_R = dx_{R,1} = dx_{R,2}}$$

$$\mathbf{A} = \left[\begin{array}{c|c|c} \mathbf{l}_{R,1} & \mathbf{l}_{O,1} & \mathbf{0} \\ \hline \mathbf{l}_{R,2} & \mathbf{0} & \mathbf{l}_{O,2} \end{array}\right]$$

Result of a Least Squares Adjustment

$$(^*) \qquad \mathbf{x(l_1, l_2) = x^0 + dx} \qquad \mathbf{dx} = \begin{bmatrix} \mathbf{dx_R} \\ \mathbf{dx_{O,1}} \\ \mathbf{dx_{O,2}} \end{bmatrix} = (\mathbf{A}^T \mathbf{C}_l^{+} \mathbf{A})^{-1} \cdot \mathbf{A}^T \mathbf{C}_l^{+} (\mathbf{l} - \mathbf{l}^0)$$

$$"\mathbf{C_l}" = \begin{bmatrix} \mathbf{C_{x1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{C_{x2}} \end{bmatrix}$$

Observation based geodetic monitoring

Coordinate-related geodetic monitoring (MONIKA)

Resulting Covariance Matrix $\mathbf{C_x}$:

$$\begin{bmatrix} C_{R,R} & C_{R,O1} & C_{R,O2} \\ C_{O1,R} & C_{O1,O1} & C_{O1,O2} \\ C_{O2,R} & C_{O2,O1} & C_{O2,O2} \end{bmatrix} = \begin{bmatrix} A_R{}^T C_l^{-1} A_R & A_R{}^T C_l^{-1} A_{O1} & A_R{}^T C_l^{-1} A_{O2} \\ A_{O1}{}^T C_l^{-1} A_R & A_{O1}{}^T C_l^{-1} A_{O1} & \\ A_{O2}{}^T C_l^{-1} A_R & & A_{O2}{}^T C_l^{-1} A_{O2} \end{bmatrix}^{-1}$$

Remark: The results $\mathbf{x}$ and $\mathbf{C_x}$ of an observation and a coordinate related approach are identical

Coordinate-related geodetic monitoring (MONIKA)

GNSS-Reference-Stations-Coordinate ***MONI***toring ***KA*** Model ***MONIKA***

MONIKA
ITRF-embedded
Deformation
Analysis Concept

RINEX

$\mathbf{l}(t_i), \mathbf{C}_l(t_i)$

$\overline{x}(t_i)_j, \overline{C}_x(t_i)_j$

$\overline{x}(t_i), \overline{C}_x(t_i)$

Epoch States
+ Geodyn. Trafo
MONIKA Step 1/2

MONIKA
Free GNSS-network
Deformation
Analysis Concept

Singular

Unknown
Datum

MONIKA
Software /System
www.monika.ag

Coordinate-related geodetic monitoring (MONIKA)

MONIKA

Free GNSS-network

Deformation Analysis Concept

Original GNSS observations

RINEX

$$\mathbf{l}(t)_i, C_l(t)_i$$

SINEX

$$\bar{x}(t_i), \bar{C}_x(t_i)$$

Epoch States

MONIKA Step 2

Singular

x_0 ?

Unknown Datum

$$\bar{x}(t_i), \bar{C}_x(t_i)$$

New x_0

Helmert-Transformation d=3

$$x(t_i) =: \bar{x}(t_i) \,\big|\, x_0$$

S-Transformation d=3

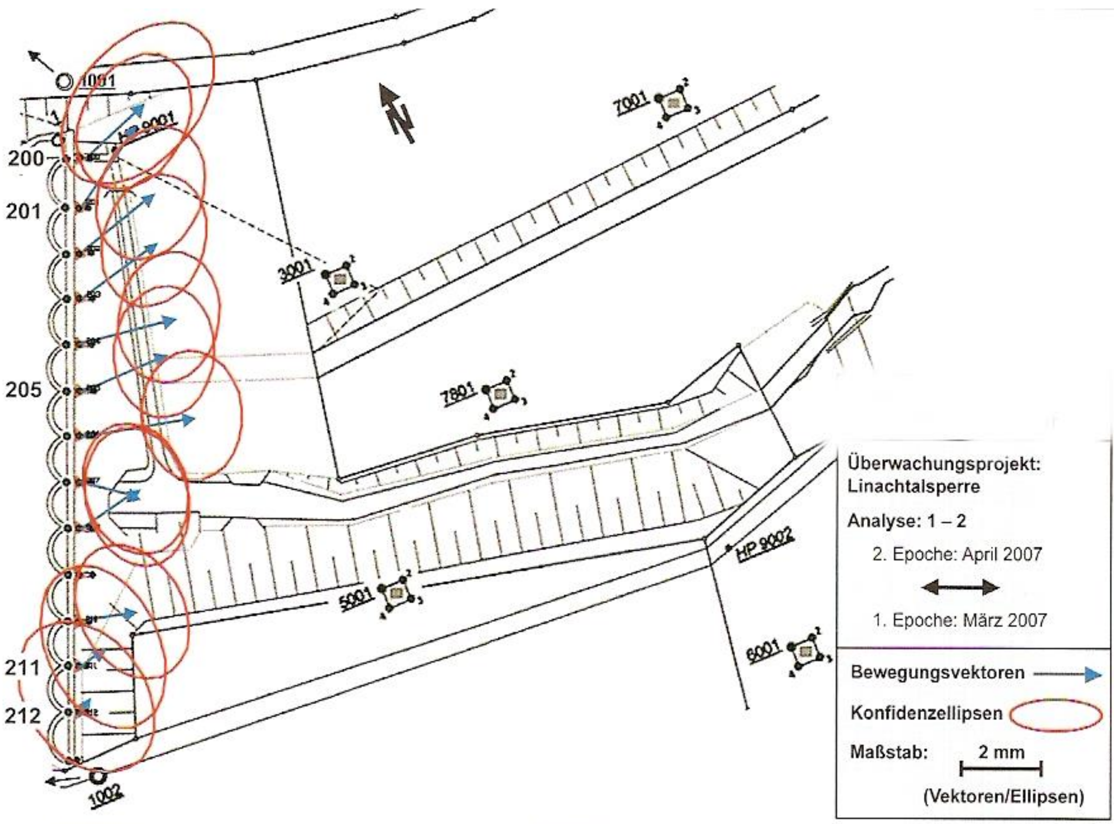
$$C_x(t_i) = S \cdot \bar{C}_x(t_i) \cdot S^T$$

MONIKA Trafo 2 - Step 2

www.monika.ag

Coordinate-related geodetic monitoring (CODEKA1D/2D)

Absolute Deformation Network
Objectpoint-Displacments and
Coordinate Related Appraoch



pekte festgelegten Standorte für die Referenzpunkte aller Voraussicht nach langfristig ihre Lage nicht verändern und die relativ zu den Referenzpunkten geschätzten Änderungen in den Objektpunktkoordinaten die Deformation des Überwachungsobjektes zwischen entsprechenden Messepochen repräsentieren.

Die grundlegende Problematik bei der Berechnung von Deformationsanalysen liegt darin, dass sich aufgrund der Stochastizität der durchgeführten Beobachtungen die geschätzten Epochenkoordinaten im Rahmen der Messgenauigkeit unterscheiden werden, selbst wenn keine realen Deformationen vorliegen. Hier gilt es also durch Anwendung statistischer Tests zwischen vorliegendem Messrauschen und eigentlicher Deformation zu unterscheiden.

Das hier zur Anwendung kommende koordinatenbezogene Verfahren zur Deformationsanalyse geht von den Ergebnissen x_i (Koordinaten) und $C_{x,i}$ (Kovarianzmatrizen) der Einzelepochenausgleichungen aus. Es bietet gegenüber den beobachtungsbezogenen Verfahren der Deformationsanalyse den Vorteil, dass lediglich die Endergebnisse der Einzelepochenausgleichungen nicht aber das hierzu verwendete Beobachtungsmaterial für die nachfolgenden Analysen zu archivieren sind. Nachfolgend wird kurz auf die einzelnen Stufen des Analysekonzepts eingegangen, wie es in Karlsruhe in dem Softwarepaket CODEKA2D umgesetzt und z. B. in (Jäger et al. 2005) näher beschrieben ist. Dabei werden nur die Schritte der Deformationsanalyse näher erläutert, auf die bei Vorstellung der Beispiele in Kap. 5.2 Bezug genommen wird.

Modeling Standards in Geodetic Monitoring (GOCA)

- **Adjustment – Step1: Initialization = Determination of Reference Points**

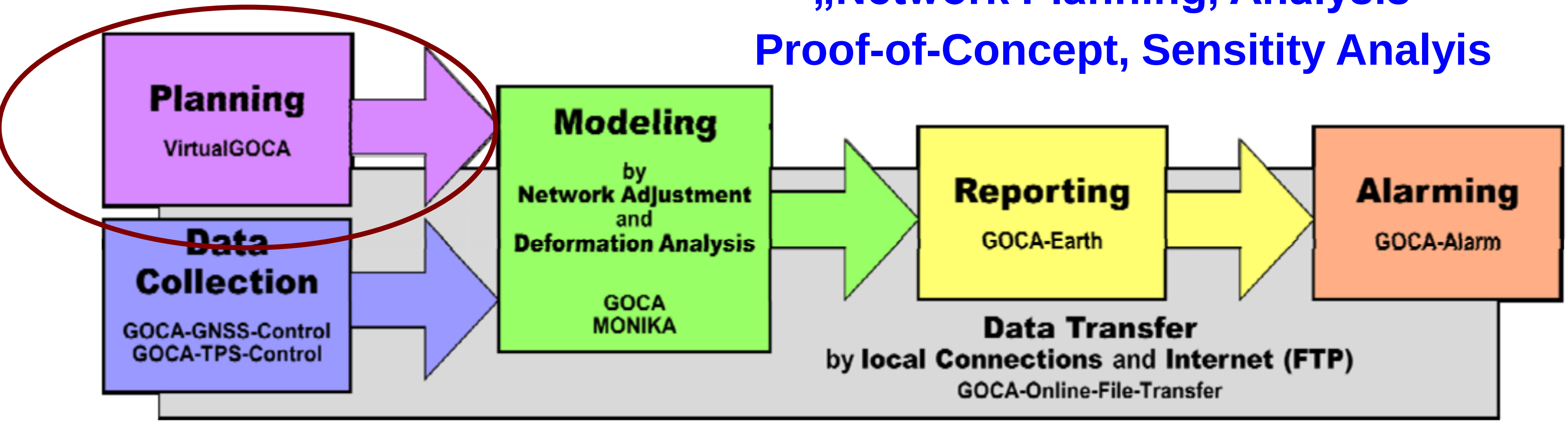
  * Related additional Feature: Stastical Control of Stability of Referencepoints by means of Adjustment Techniques

- **Adjustment - Step 2: Continuous Adjustment of Object Point Positions in the Reference Point Datum and Visualization of Objectpoint Time Series in Graphical Window.**

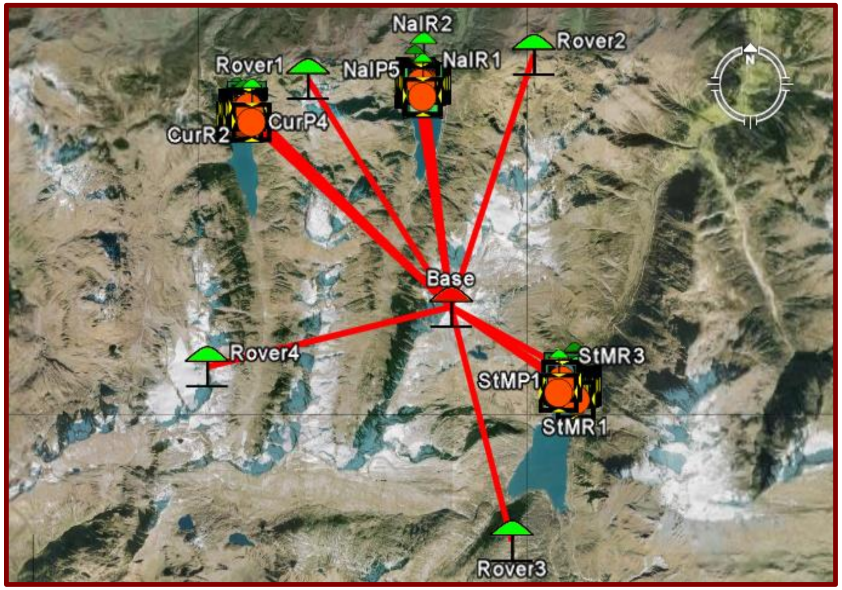
- **Adjustment - Step 3 : GOCA - Deformationanalysis**
  - Online Moving Average Displacemets
  - Online Displacement Estimation, Statistical Testing and Alarm Setting
  - Online-Estimation of Displacement, Velocity and Acceleration based on a Kalman-Filtering. Computation of Alarm Probability for each Object Point.

Modeling Standards in Geodetic Monitoring (GOCA)

„Network Planning, Analysis“
Proof-of-Concept, Sensitity Analyis



<u>Virtual-GOCA Software</u>
Generates <u>Sensor Data</u>
<u>in Geometry & Gravity Space</u> for
GNSS, Totalstations and Leveling
Purposes:
„Proof-of-Concept“ of designed projects and SW-Development



Trends
in
Geodetic Monitoring

FIG Commission 6, Trend 1 - Integrated Deformation-Analysis
- FEM Approaches -

System-Equations

$K \cdot u = f$

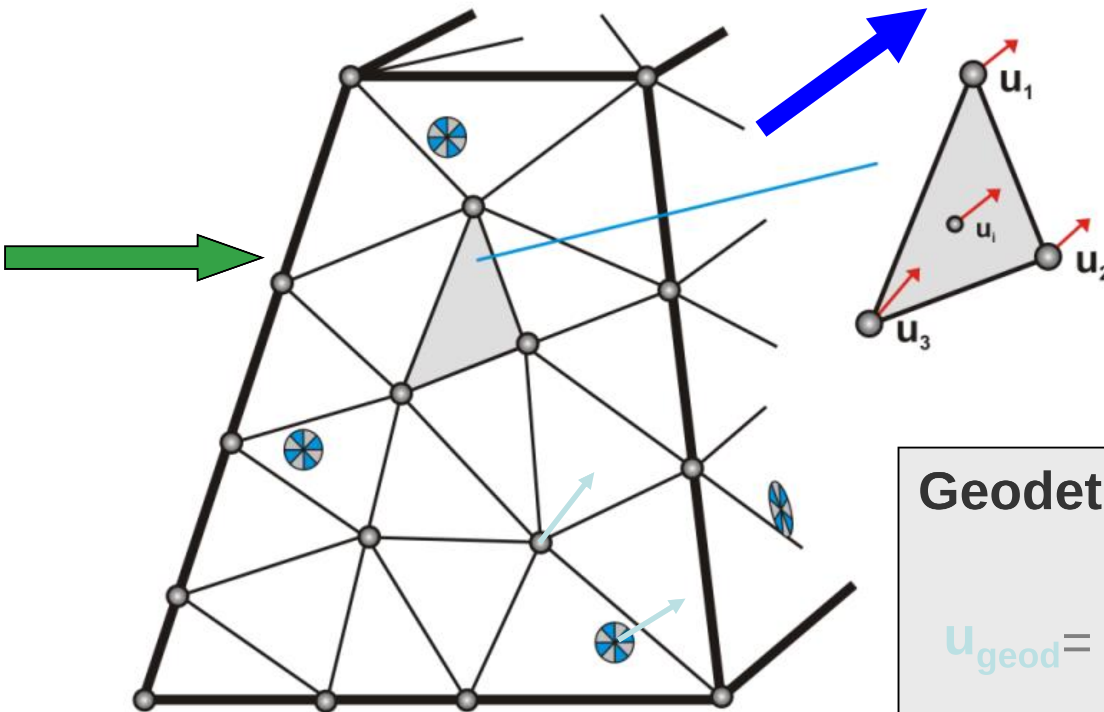
$K(p, \Delta p)$

Stiffness Matrix

p = Material Parameters

Δp= Changes (Damage, Fissure)

FEM-Elements



$$\mathbf{u}_{\mathbf{i},\mathrm{E}} = \mathbf{N}_{\mathbf{E}} \cdot \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \end{pmatrix} = \mathbf{N}_{\mathbf{E}} \cdot \mathbf{u}_{\mathrm{E-Nodes}}$$

N_E=Shape-Functions

Knoten des geodätischen Netzes

Knoten des FE-Modells

Geodetic Displacements u_{geod}

$$u_{geod} = \mathbf{N}_{\mathbf{E}}(\mathbf{x}_{\mathrm{geod}}) \cdot \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \end{pmatrix} = \mathbf{N}_{\mathbf{E}}(\mathbf{x}_{\mathrm{geod}}) \cdot \mathbf{E} \cdot u$$

E= (0,1)-Matrix => Geod. Design (FOD)

FIG Commission 6, Trend 1 - Integrated Deformation-Analysis – STATIC Approach and Sensor-Integration

FEM- / System- Parts (Static) & *Parameter-Integration*

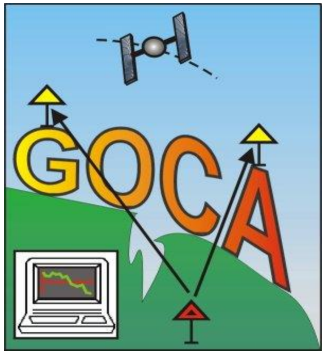
$$\mathbf{0}_{sys} + \mathbf{v}_{sys} = \hat{\mathbf{u}} - \mathbf{K}(\hat{\mathbf{p}}_k, \Delta\hat{\mathbf{p}})^{-1} \cdot \hat{\mathbf{f}}$$ and $\mathbf{C}_{sys}$ => $\mathbf{0}$

$$\mathbf{p}_k + \mathbf{v}_p = \hat{\mathbf{p}}_k$$ und $\mathbf{C}_{p_k}$

$$\mathbf{f} + \mathbf{v}_f = \hat{\mathbf{f}}$$ und $\mathbf{C}_f$

Geod. Monitoring & Network Adjustment

$$\mathbf{u}_{geod} + \mathbf{v}_{geod} = \mathbf{N}_E \cdot \mathbf{E}_{geod}\, \hat{\mathbf{u}}$$ and $\mathbf{C}_{u,geod}$



Sensor-Integration

$$\mathbf{l}_{geom} + \mathbf{v}_{geom} = \mathbf{l}_{geom}(\mathbf{N}_E, \mathbf{E}_{geom}, \hat{\mathbf{u}})$$ and $\mathbf{C}_{geom}$

Local Extensiometers, Strain

$$\boldsymbol{\varepsilon} = \mathbf{L} \cdot \mathbf{N}_E \cdot \mathbf{E}_{geom} \cdot \hat{\mathbf{u}}$$

$$\mathbf{l}_{phys} + \mathbf{v}_{phys} = \mathbf{l}_{phys}(\mathbf{N}_E, \mathbf{E}_{phys}, \hat{\mathbf{u}}, \hat{\mathbf{p}}_k, \Delta\hat{\mathbf{p}})$$

and $\mathbf{C}_{phys}$

Local pressure, stress

$$\boldsymbol{\sigma} = \mathbf{D}(\mathbf{p}_k) \cdot \mathbf{L} \cdot \mathbf{N}_E \cdot \mathbf{E}_{phys} \cdot \hat{\mathbf{u}}$$

FIG Commission 6, Trend 1 - Integrated Deformation-Analysis – DYNAMIC Approach and Sensor-Integration

General Vibration of a Structure in FEM Dynamics

$$\mathbf{K}(\mathbf{p}_K)\cdot\mathbf{u}(t)+\mathbf{C}(\mathbf{p}_C)\cdot\dot{\mathbf{u}}(t)+\mathbf{M}(\mathbf{p}_M)\cdot\ddot{\mathbf{u}}(t)=\mathbf{f}(t)$$

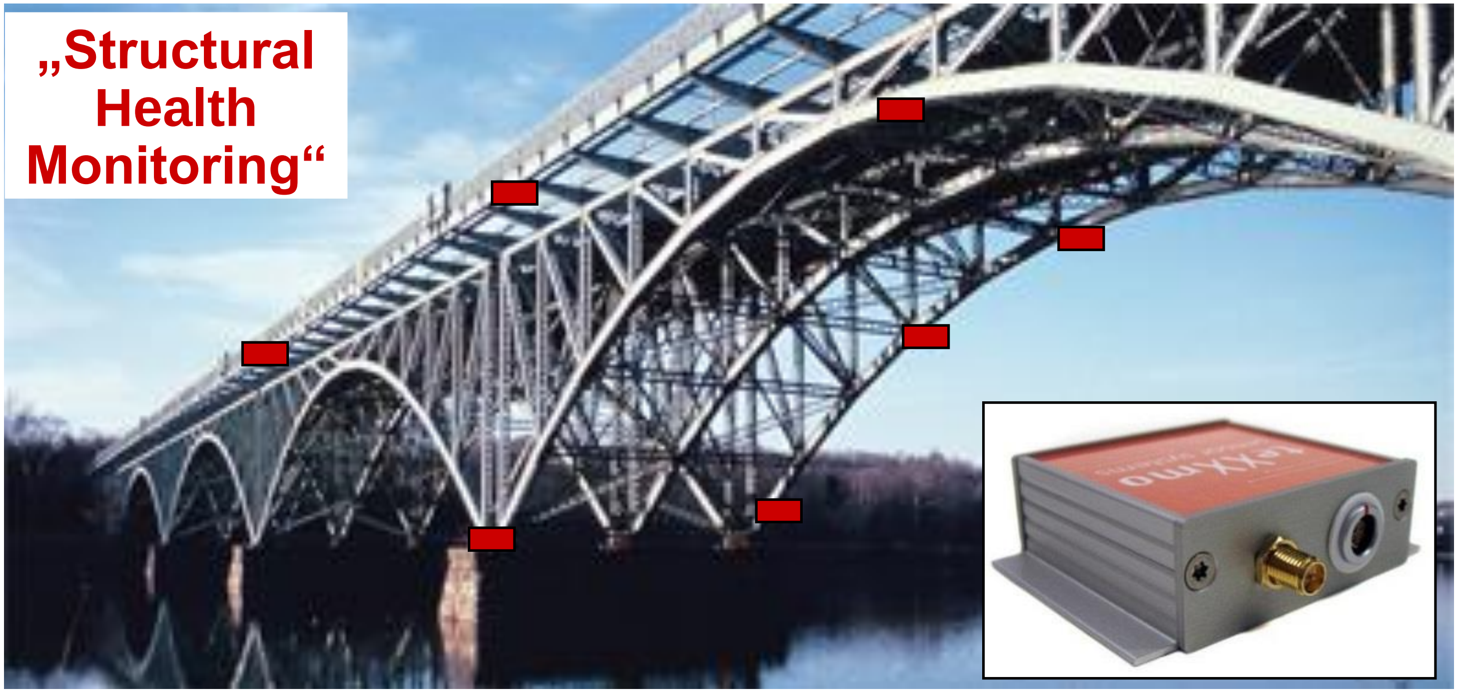
Eigenvibration of Structures in FEM Dynamics

$$\mathbf{K}(\mathbf{p}_K)\cdot\mathbf{u}(t)+\mathbf{C}(\mathbf{p}_C)\cdot\dot{\mathbf{u}}(t)+\mathbf{M}(\mathbf{p}_M)\cdot\ddot{\mathbf{u}}(t)=\mathbf{0}$$

$K(p_K)$ = Parametrized Stiffnessmatrix
$C(p_C)$ = Parametrized Damping-Matrix
$M(p_M)$ = Parametrized Mass-Matrix
$f(t)$ = External Nodal Point Force

Research Topic in „System Analysis“ or „Integrated Deformation Analysis“

General Trend 2
– New Sensors of all kind and disciplines, including new geodetic sensors -



„Marriage“/Integration of Geodetic Monitoring & Navigation-Algorithms/-Sensors

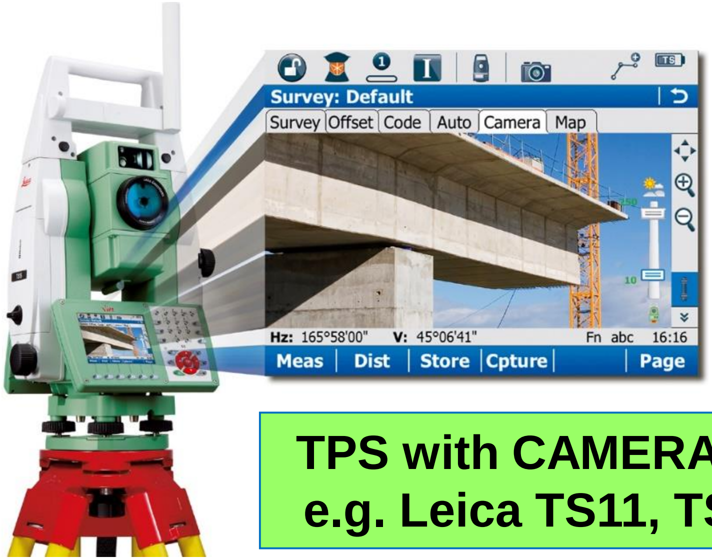
GOCA

NAVKA www.navka.de

$$y(t) = \left[x^e \; y^e \; z^e \,|\, \dot{x}^e \dot{y}^e \dot{z}^e \;|\, r^e \; p^e \; y^e \,\|\, \ddot{x}^e \; \ddot{y}^e \; \ddot{z}^e \,|\, \omega^b_{eb,x} \; \omega^b_{eb,y} \; \omega^b_{eb,z} \,|\, \mathbf{s} \right]^T$$

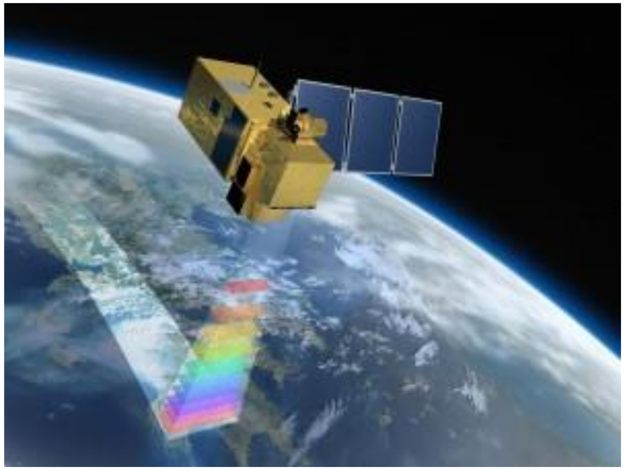
General Trend 2
– New Sensors of all kind and disciplines, including new geodetic sensors -

LASERSCANNERS e.g. Faro



TPS with CAMERA Sensors e.g. Leica TS11, TS15, TS50

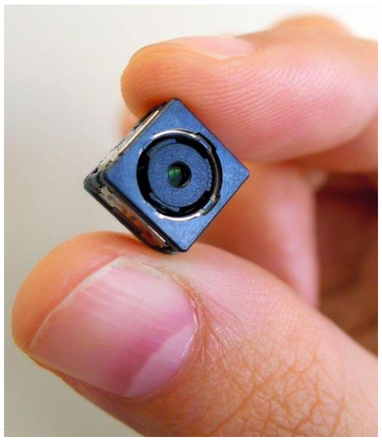
Terrestrial LIDAR, e.g. Optech

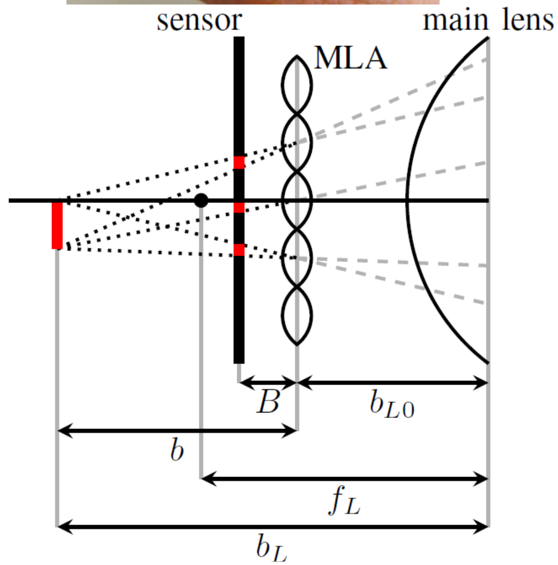


Space-borne LIDAR e.g. ESA Sentinel

General Trend 2
– New Sensors of all kind and disciplines, including new geodetic sensors -

Plenoptic Cameras





Zenith Cameras Geodynamic Networks

Gravity Meters Geodynamic Networks

Parameter Estimation
in
Geodetic Monitoring
in
Geometry & Gravity Space

Standards ?

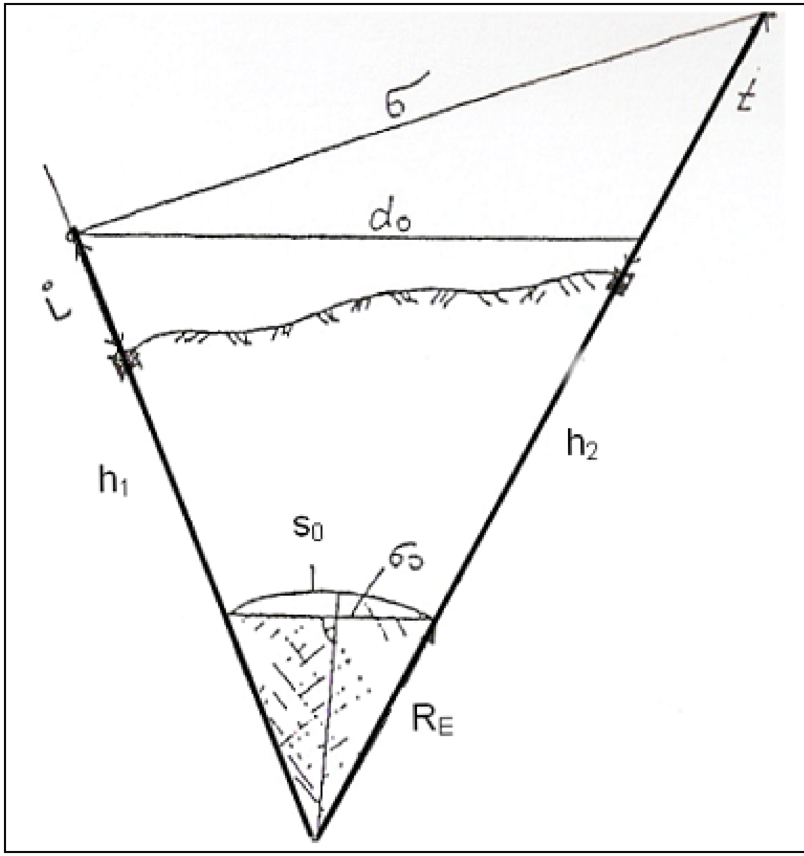
2D/1D Network Approaches

Network Unknowns
Parameters
(N,E,h) or (N,E,H)

Sensor Network Geometric Space - 1D/2D Parametrization

2D/1D by reduction of the measurements to the ellipsoid ("plan") and to the physical or ellipsoidal height system ("height")

3D as "2D/1D-approach". Strict functional modelling, only a negligence of the correlations between the resulting plan and height components



TPS-observations

z = zenith angle observation

σ = slope distance observation

r = direction observation

Sensor Network Geometric Space - 1D/2D Parametrization

2D - Horizontal distances (iteratively)

Start: $d_0 = \sigma \cdot \sin z$

Refraction angle: $\delta = \dfrac{k \cdot \sigma}{2 \cdot R_E}$

1.] $$\varepsilon = \frac{d_0}{2 \cdot (R_E + h_1 + i)}$$

2.] $$d_0 = \sigma \cdot \frac{\sin(z + \delta - 2\varepsilon)}{\cos\varepsilon}$$

Next: $$\sigma_0 = d_0 - \frac{d_0 \cdot (h_1 + i)}{R_E}$$ and

$$s_0 = \sigma_0 + \frac{\sigma_0^{\,3}}{24 \cdot R_E^{\,2}}$$

Finally:

$s_{Pr\,ojected}$ - for 2D adjustment

1D- Height difference (physical heights H)

$$\Delta H = H_2 - H_1 = \sigma \cdot \frac{\cos(z + \delta - \varepsilon)}{\cos\varepsilon} + i - t$$

Height difference (ellips heights)

$$\Delta h = h_2 - h_1 = \sigma \cdot \frac{\cos(z_{red} + \delta - \varepsilon)}{\cos\varepsilon} + i - t$$

Reductions

- Reduction of direction measurements r due to vertical deflections (ς, η)
- Reduction (ς, η) of zenith angle measurements z leads to unification of heights reference frame

Sensor Network Geometric Space - 1D/2D Parametrization

Absolute Deformation Network $y = (x_R, x_o(t))$
<u>Geodetic Network Adjustment</u>

Standard Sensors l=l(x)
GPS/GNSS , Total Stations
Optical and Hydrostatic Levels

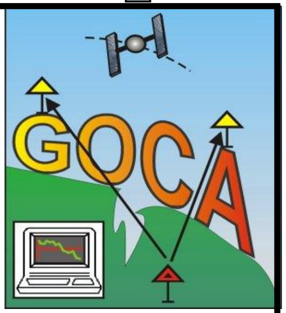
Sensor-
Integration

Common Adjustment of redundant sensor observations l(x,t): Unique set of 3D-coordinates per object point and per epoch t

$$\Delta Ho_{ij,GPS}(t) + v = \Delta \hat{H}o_{ij}(t)$$

$$\Delta Re_{ij,GPS}(t) + v = \Delta \hat{R}e_{ij}(t)$$

$$\Delta h_{ij,GPS}(t) + v = \Delta \hat{h}_{ij}(t)$$



$$r_{ij}(t) + v = \arctan\left(\frac{\Delta \hat{R}_{ij}}{\Delta \hat{H}_{ij}}\right)^t - \hat{o}$$

$$s_{ij}(t) + v = \left(\hat{m} \cdot \sqrt{\Delta \hat{R}_{ij}^2 + \Delta \hat{H}_{ij}^2}\right)^t$$

$$\Delta H_{ij}(t) + v = \Delta \hat{h}_{ij}(t) + \hat{A} \cdot \Delta Re_{init} + \hat{B} \cdot \Delta Ho_{Init} + \Delta \hat{m}_h \cdot \Delta h$$

3D Quasi-Integrated Geometry and Gravity Field Approach

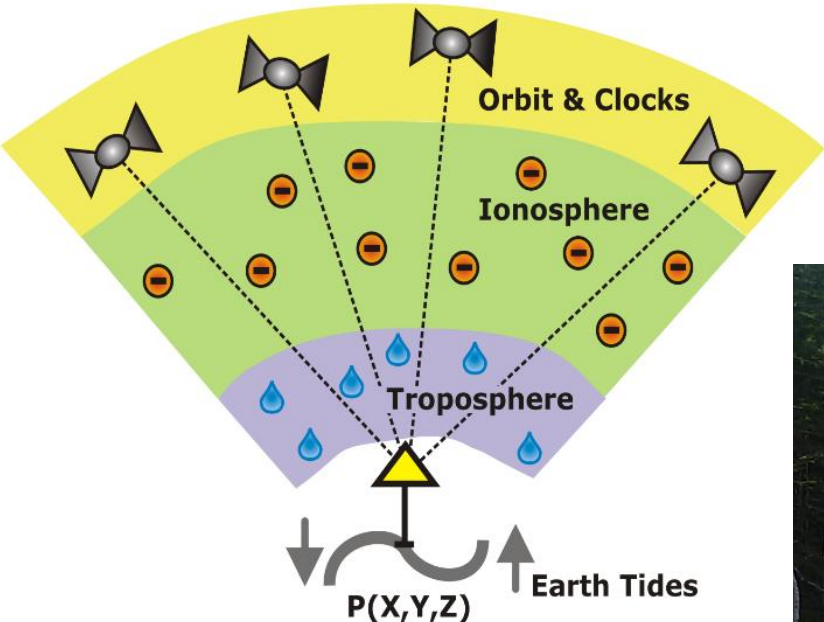
Network Unknowns / Parameters

Geocentric Cartesian Coordinates (x,y.z)

- **Gravity Field W (C_{nm},S_{nm}) as**
- **„Auxiliary Stochatical Quantity“**

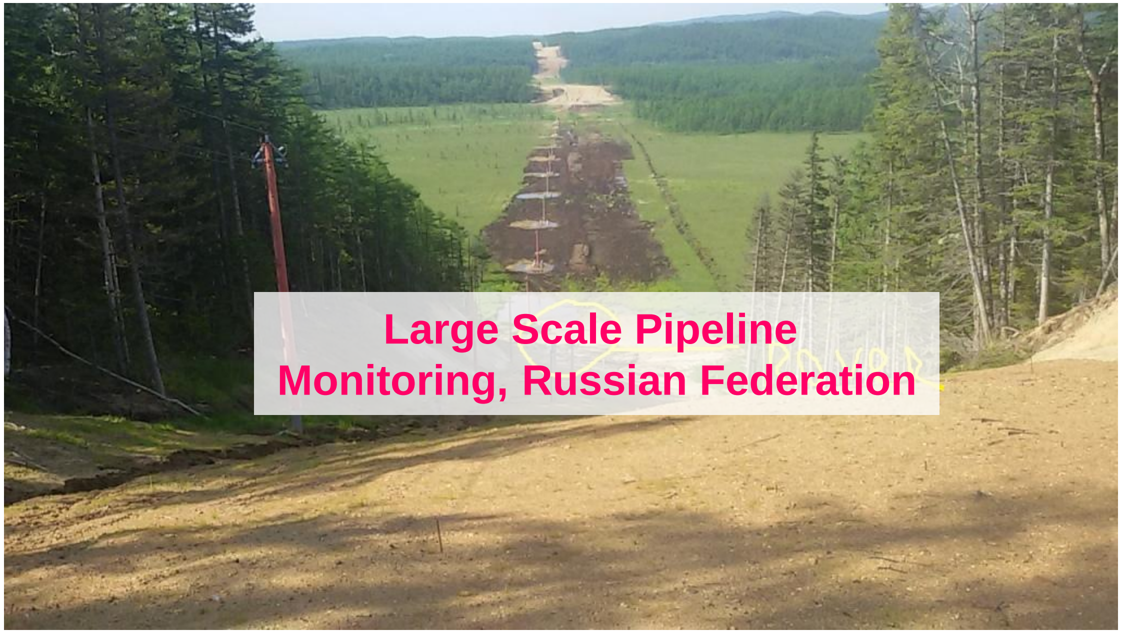
Sensor Network Geometric Space – 3D Parametrization in Q-Integrated Approach

GNSS-Positions – necessary to be integated in „OPPP age“ and „GNSS/MEMS sensor fusion age“.

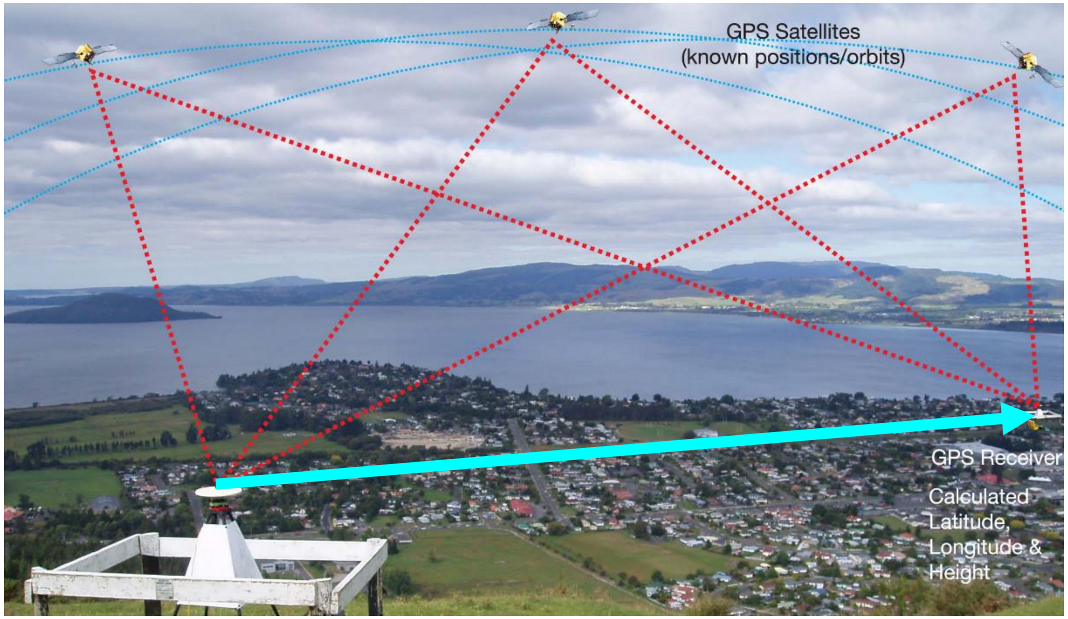


$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Useful for extended gemonitoring scenarios, e.g. large pipeline networks



Sensor Network Geometric Space
– 3D Parametrization in Q-Integrated Approach



GNSS-Baselines

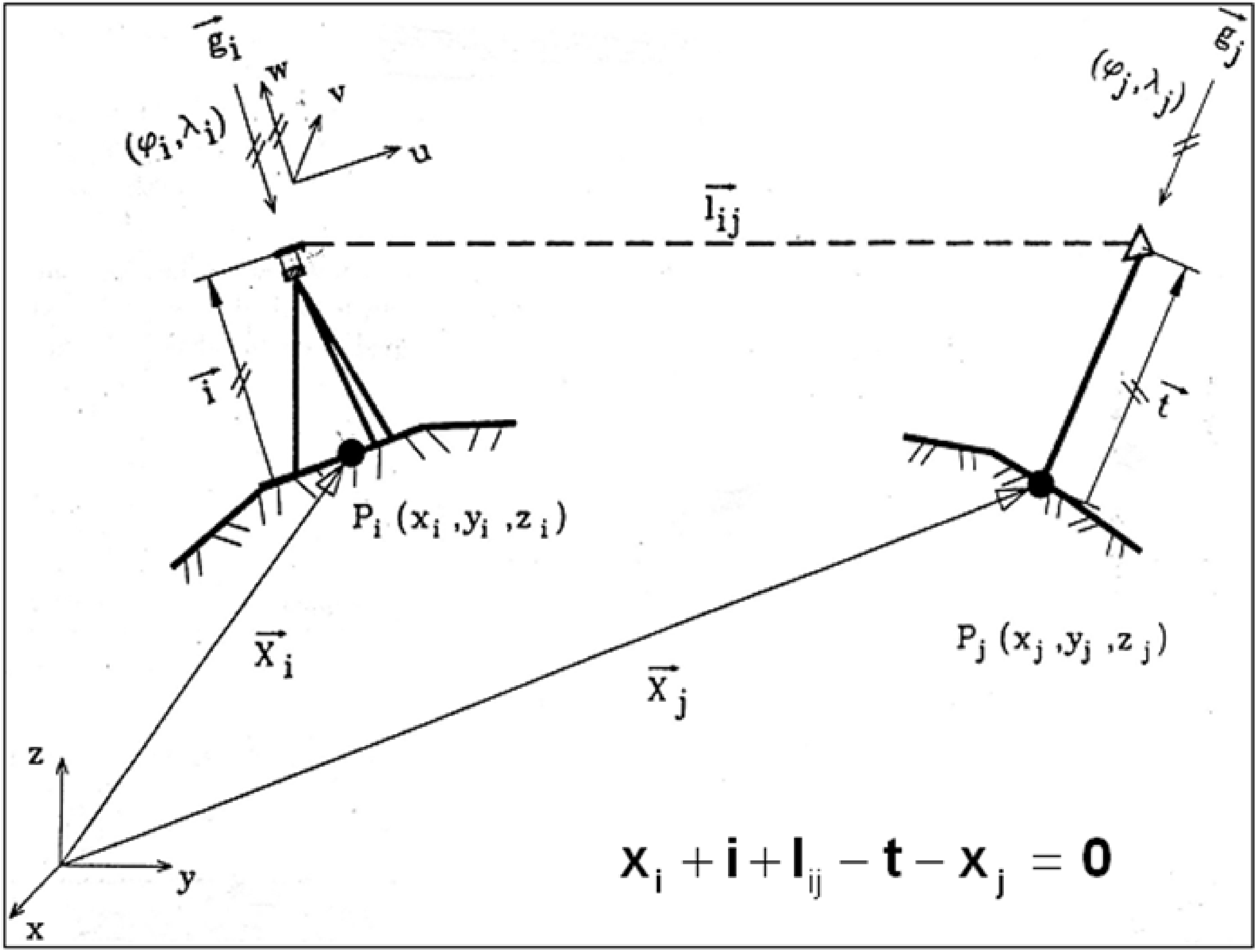
$$\mathbf{b} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \mathbf{x}_{Rover} - \mathbf{x}_{Base}$$

Stochastic behaviour
Covariance Matrix of Baselinevectors b

$$\mathbf{C_b} = \mathbf{F} \cdot \begin{pmatrix} (\sigma_N/(M+h))^2 & 0 & 0 \\ 0 & (\sigma_E/((N+h)\cdot\cos B)^2 & 0 \\ 0 & 0 & \sigma_h^2 \end{pmatrix} \cdot \mathbf{F}^T$$

$$\mathbf{F} = \begin{bmatrix} -(M+h)\cdot\sin B\cdot\cos L & -(N+h)\cdot\cos B\cdot\sin L & \cos B\cdot\cos L \\ -(M+h)\cdot\sin B\cdot\sin L & (N+h)\cdot\cos B\cdot\cos L & \cos B\cdot\sin L \\ (M+h)\cdot\cos B & 0 & \sin B \end{bmatrix}$$

Sensor Network Geometric Space – 3D Parametrization in Q-Integrated Approach



Total Stations

5 unknowns per point

Sensor Network Geometric Space – 3D Parametrization in Q-Integrated Approach

$$\mathbf{l}_{ij}^{LAV,i} = \mathbf{D}_i(\varphi_i,\lambda_i)\cdot\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix}_{ij}^{ECEF} + \begin{bmatrix}0\\ 0\\ -i\end{bmatrix}^{LAV,i} + \mathbf{D}_i(\varphi_i,\lambda_i)\cdot\mathbf{D}_j(\varphi_j,\lambda_j)^T\cdot\begin{bmatrix}0\\ 0\\ t\end{bmatrix}^{LAV,j}$$

$$\downarrow$$

$$\mathbf{l}_{ij}^{LAV,i} = \begin{bmatrix}\Delta u\\ \Delta v\\ \Delta w\end{bmatrix}_{ij}^{LAV,i}$$

TPS Total-station

$$s_{ij} = \sqrt{\Delta u^2 + \Delta v^2 + \Delta w^2}$$

3D-Slope Distance

$$r_{ij} = \arctan(\frac{\Delta v}{\Delta u}) - o_i$$

Direction

$$z_{ij} = \arctan(\frac{\sqrt{\Delta u^2 + \Delta v^2}}{\Delta w}) - \frac{s_{ij}}{2R}\cdot k$$

Zenith Angle

$$\mathbf{D}_e^{LAV} = \begin{bmatrix}-\cos\lambda\cdot\sin\varphi & -\sin\lambda\cdot\sin\varphi & \cos\varphi\\ -\sin\lambda & +\cos\lambda & 0\\ \cos\lambda\cdot\cos\varphi & \sin\lambda\cdot\cos\varphi & \sin\varphi\end{bmatrix}$$

Geopotential-Modell W (C_{nm}; S_{nm})

$$\varphi = \arctan\frac{-W_z}{\sqrt{W_x^2 + W_y^2}}$$

$$\lambda = \arctan(\frac{W_y}{W_x})$$

As direct observations taken form W

Sensor Network Geometric Space – 3D Parametrization in Q-Integrated Approach

Hydrostatic Levels and Automatic Levels

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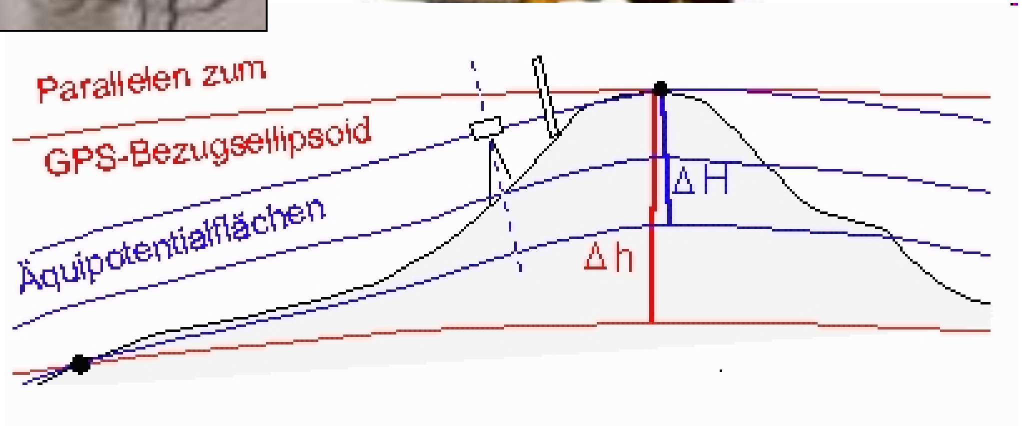
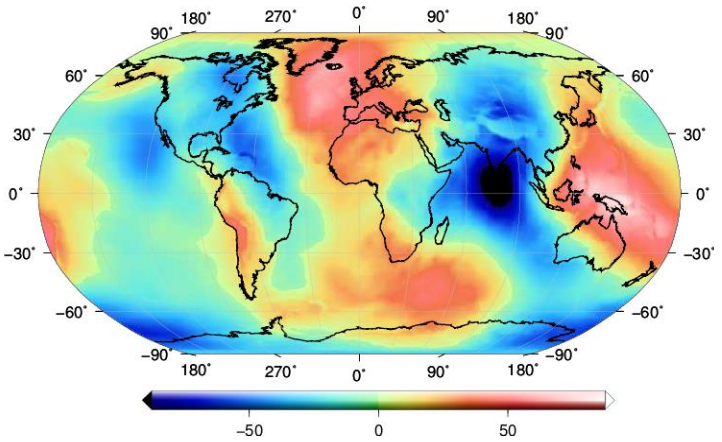
1 st Approach





Geopotential-Model W (C_{nm}; S_{nm})

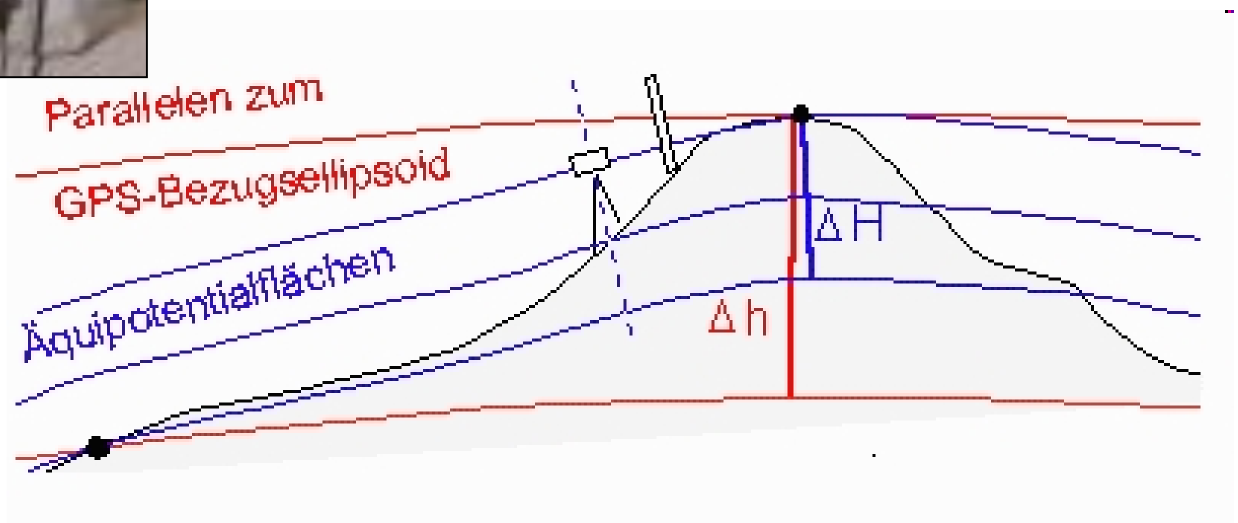
$$\Delta H_{ij} = H_j - H_i = (h_j - N_j) - (h_i - N_i) = (h_j - \left(\frac{(W-U)_P}{\gamma_Q}\right)_j) - (h_i - \left(\frac{(W-U)_P}{\gamma_Q}\right)_i)$$

Quasigeoid-Theory of Molodenski

Sensor Network Geometric Space
– 3D Parametrization in Q-Integrated Approach

Hydrostatic Levels
and
Automatic Levels

2nd Approach

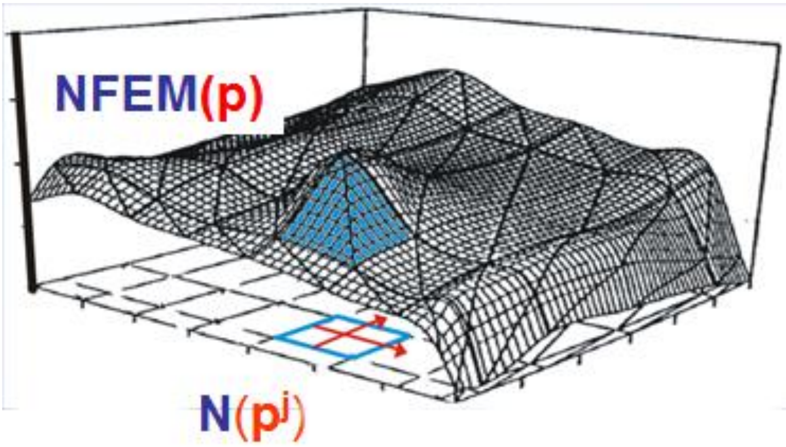


$$\Delta H_{ij} = H_j - H_i = (h_j - h_i) + [(-N \cdot e^2 \cdot \sin(B) \cdot \cos(B) \cdot \sin(L)) \cdot e_x + N \cdot e^2 \cdot \sin(B) \cdot \cos(B) \cdot \cos(L) \cdot e_y]$$

Sensor Network Geometric Space
– 3D Parametrization in Q-Integrated Approach

Hydrostatic Levels
and
Automatic Levels

3rd Approach



„DFHBF-like"

Implemented for CERN Network

$$H + v = \hat{h}(x, y, z) - NFEM(\hat{p})$$

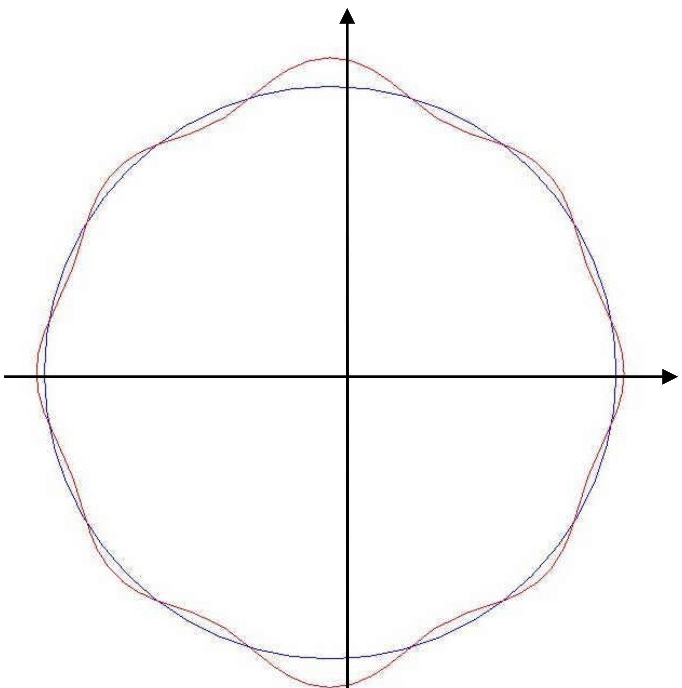
3D (Strict) Integrated Geometry and Gravity Field Approach

Network Unknowns / Parameters

- Geocentric Cartesian Coordinates(x,y.z)
- Gravity Field W by Parameters W (C_{nm},S_{nm})

Sensor Network Geometric Space
– 3D Parametrization in Integrated Geometry and Gravity Approach

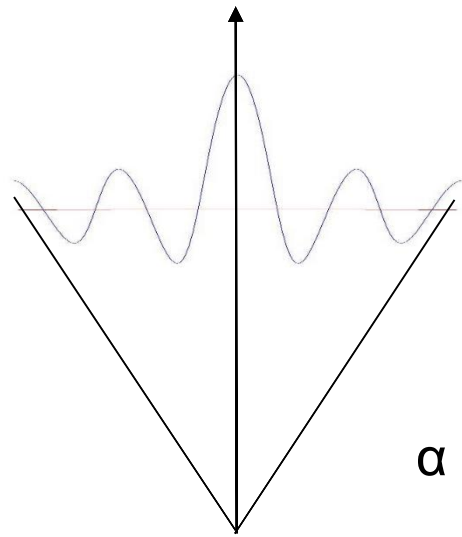
Global Spherical-Harmonics (SH)



$\alpha_0 = 90°$

SH „Resolution“ for „2mm QGeoid“
n=7200 and u =50.000.000 Coefficients

Spherical Cap Harmonics (SCHA)



α_0 = arbitrary

$$n_{SH} =\approx \frac{90°}{\alpha} \cdot (k_{,SCH} + 0.5) - 0.5$$

α = 1° (=110 km area) => k-SCH = 80 and u = 6.400

α = 3° (=330 km area) => k-SCH = 250 and u= 62.500

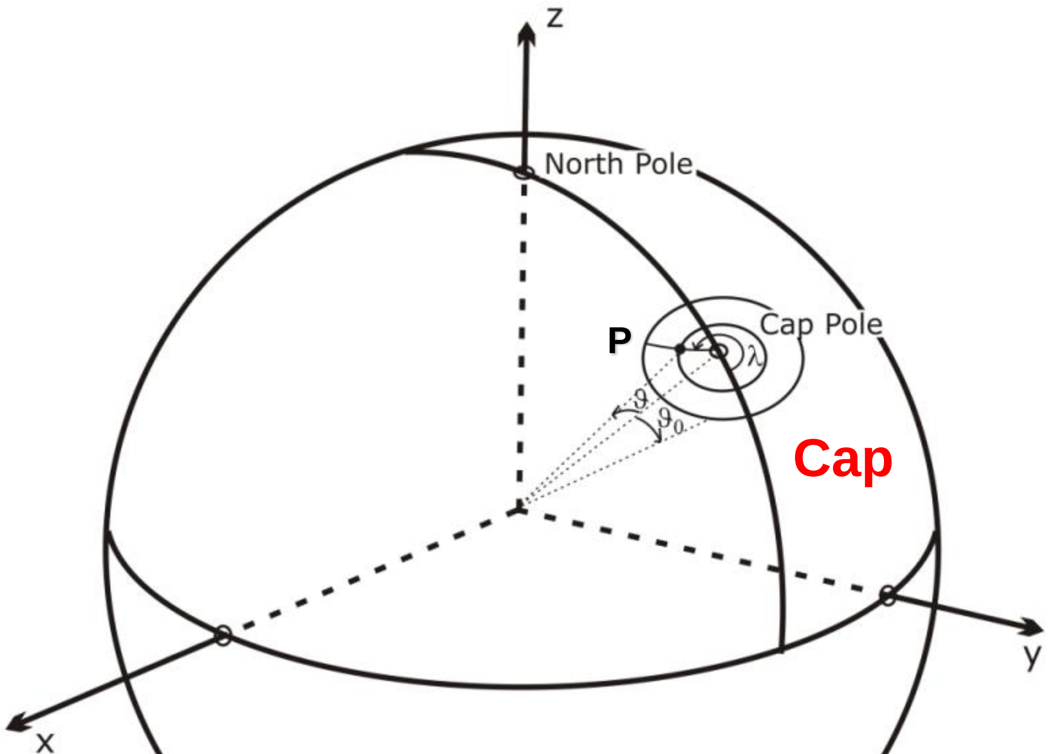
α = 25° (Europe) => k-SCH = 2000 and u =4.000.000

Sensor Network Geometric Space
– 3D Parametrization in Integrated Geometry and Gravity Approach

Sperical Cap Harmonics (SCH)

as

Carrier Functions of the Gravity Potential V



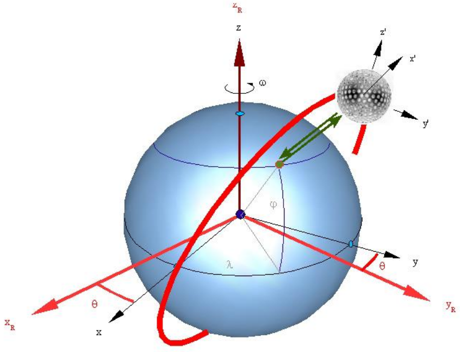
$$\tan\lambda' = \frac{\cos\varphi.\sin(\lambda-\lambda_0)}{\sin\varphi\cos\varphi_0 - \cos\varphi\sin\varphi_0\cos(\lambda-\lambda_0)}$$

$$\cos\theta' = \sin\varphi\sin\varphi_0 - \cos\varphi\cos\varphi_0\cos(\lambda-\lambda_0)$$

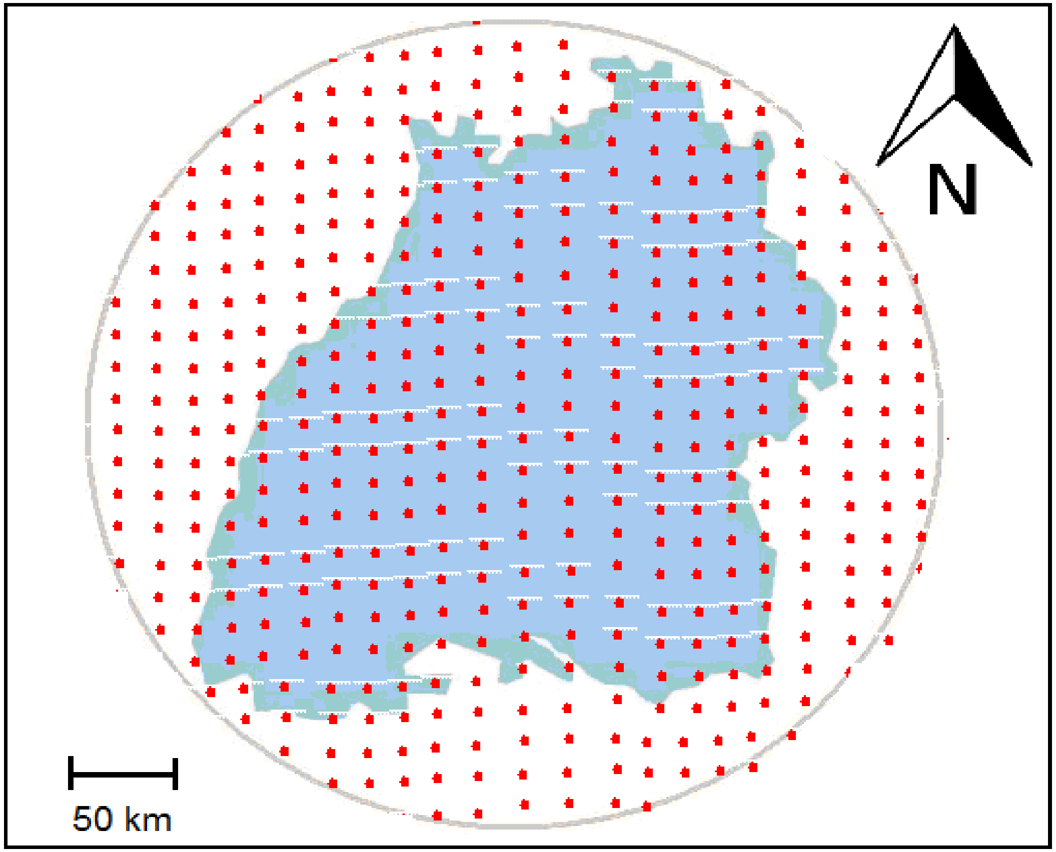
$$V(r,\lambda',\theta') = \frac{G\cdot M}{a}\sum_{k=0}^{k\max}\left(\frac{a}{r}\right)^{n(k)+1}\sum_{m=0}^{k}(C'_{n(k),m}\cdot\cos m\lambda' + S'_{n(k),m}\cdot\sin m\lambda')\cdot P_{n(k)),m}(\cos\theta'))$$

Sensor Network Geometric Space
– 3D Parametrization in Integrated Geometry and Gravity Approach

1. Treatment of Global Gravity Models – Example Baden-Württemberg



„Mapping“ of Global SCH-Coefficients (e.g. EGM 2008) (C_{nm}, S_{nm}) to local SCHA-Coefficients (C_{nm}‘, S_{nm}‘)



1.1 3D-Grid Generation

$$V_{SCHA}(r,\theta',\lambda') = \frac{GM}{r}\sum_{k=0}^{kmax}\left(\frac{R}{r}\right)^{n(k)+1}\sum_{m=0}^{k}\left(C'_{nm}\cos m\alpha' + S'_{nm}\sin m\alpha'\right)\bar{P}_{n,m}(\cos\theta') = V_{Global}(r,\theta,\lambda)$$

1.2 Computation of $(C_{nm},S_{nm})'$ from (C_{nm},S_{nm}) as Adjustment

$$\mathbf{l} = [V_1 \quad V_2 \quad V_3 \quad V_4 \quad \cdots\cdots \quad V_m]^T$$

$$\mathbf{A} = \begin{bmatrix} \frac{GM}{r_1} & \frac{GM}{r_1}\frac{R}{r_1}P_{10} & \frac{GM}{r_1}\frac{R}{r_1}\cos\alpha_1 P_{11} & \frac{GM}{r_1}\frac{R}{r_1}\sin\alpha_1 P_{10} & & \\ \frac{GM}{r_2} & \frac{GM}{r_2}\frac{R}{r_2}P_{10} & \frac{GM}{r_2}\frac{R}{r_2}\cos\alpha_2 P_{11} & \frac{GM}{r_2}\frac{R}{r_2}\sin\alpha_2 P_{10} & \cdots & \cdots \\ \frac{GM}{r_3} & \frac{GM}{r_3}\frac{R}{r_3}P_{10} & \frac{GM}{r_3}\frac{R}{r_3}\cos\alpha_3 P_{11} & \frac{GM}{r_3}\frac{R}{r_3}\sin\alpha_3 P_{10} & & \\ & \vdots & & & \ddots & \\ & \vdots & & & & \\ & \vdots & & & & \frac{GM}{r_m}\left(\frac{R}{r_m}\right)^n \sin m\alpha_m P_{nn} \end{bmatrix}$$

$$\hat{\mathbf{x}} = [C'_{00} \quad C'_{10} \quad C'_{11} \quad S'_{11} \quad C'_{20} \quad \cdots\cdots \quad S'_{nn}]^T$$

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{l}$$

Sensor Network Geometric Space – 3D Parametrization in Integrated Geometry and Gravity Approach

Treatment of Gravity Observations

$$\begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} = \mathbf{g}^{LAV}$$

Sensor-Observation at Position P(x,y,z)

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}^{ECF} = R(B, L, \eta, \xi)_{LAV}^{ECF} \cdot \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}^{LAV} \Rightarrow \begin{bmatrix} g_x' \\ g_y' \\ g_z' \end{bmatrix}_{grav}^{ECF} = \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}^{ECF} - \left[g_{centrifugal}^{ECF} \right]$$

$$\Rightarrow \begin{bmatrix} g_N \\ g_E \\ g_r \end{bmatrix}_{grav}^{LGV} = R(\varphi, \lambda)_{ECF}^{LGV} \cdot g'^{ECF}_{grav} \qquad \mathbf{g}_{grav}^{LGV} = \left[\frac{1}{r} \cdot \frac{\partial V}{\partial \theta'}, \frac{1}{r \cdot \sin\theta'} \cdot \frac{\partial V}{\partial \lambda}, \frac{\partial V}{\partial r}\right]^T$$

$$g_{grav_r}^{LGV} + v = \sum_{k=0}^{\infty} \left(\frac{a}{r}\right)^{n(k)+1} \frac{(n(k)+1)}{r} \sum_{m=0}^{k} (\bar{C}'_{n(k)),m} \cdot \cos m\lambda' + \bar{S}'_{n(k),m} \cdot \sin m\lambda') \cdot P_{n(k),m}(\cos\theta')$$

Sensor Network Geometric Space – 3D Parametrization in Integrated Geometry and Gravity Approach

Used Data - Input DFHBF 5.0

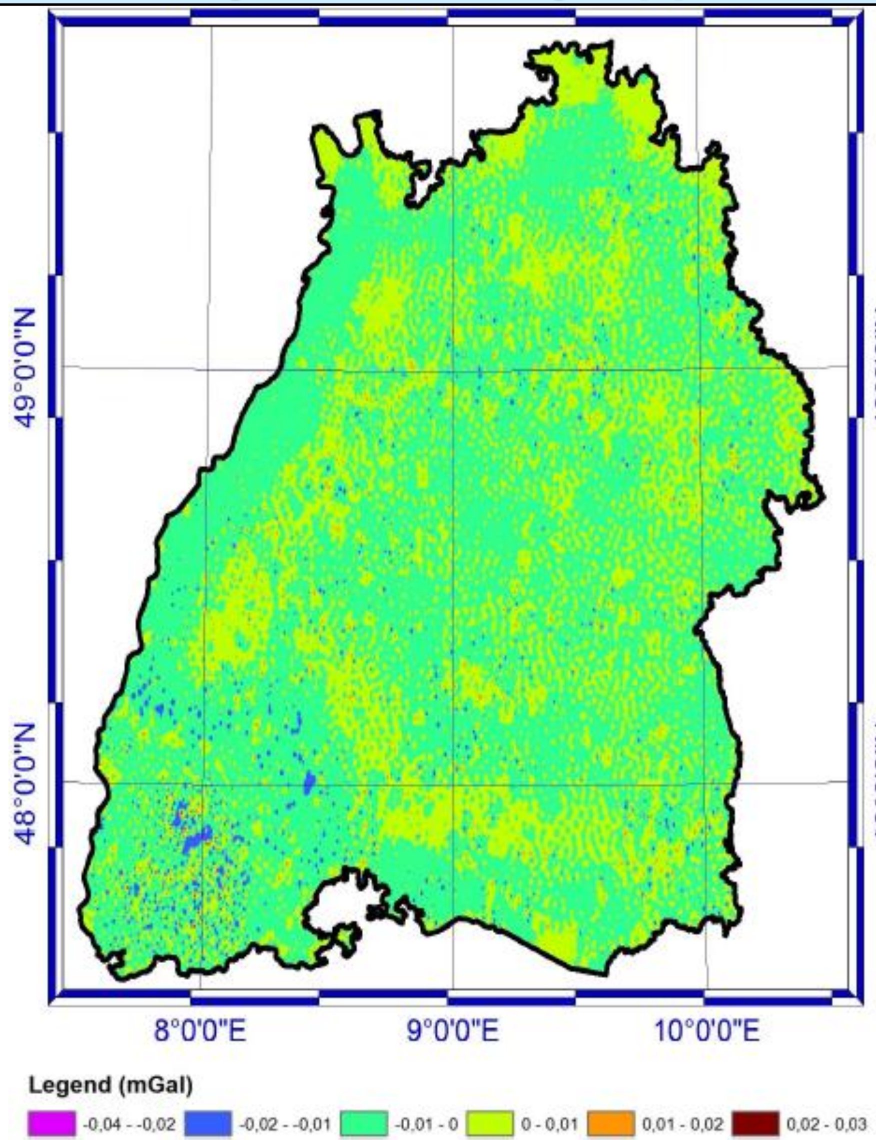
- 129 Fitting Points (B,L,h|H)
- 13671 Gravity Values (final)
- EGM2008 Global GPM mapped to SCH

• SCH Degree (m,n(k)) = 300

2. Output DFHBF 5.0)

Terrestrial Gravity Points g(B,L,h)

- Mean Residual: 0.0032 mGal
- 1031 gravity values eliminated in advance due to wrong georeferencing
- 269 blunders detected in DFHBF V. 5.0 adjustment by datasnooping

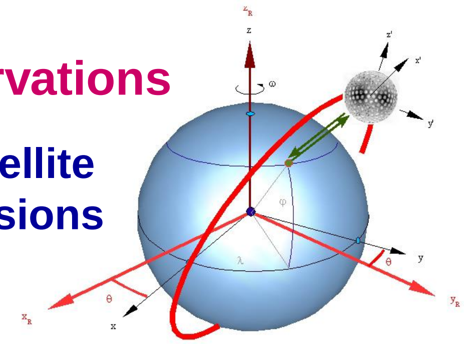


Sensor Network Geometric Space
– 3D Parametrization in Integrated Geometry and Gravity Approach

1. SCHA-Coefficients computed from global SH as direct observations

$$C'_{n(k),m}(t) + v = \hat{C}'_{n(k),m} \quad \text{and} \quad S'_{n(k),m}(t) + v = \hat{S}'_{n(k),m}$$

Satellite Missions



2. Gravity Observations rotated to SCHA

Gravity-Meter

$$g^{LGV}_{grav\ r} + v = \sum_{k=0}^{\infty} \left(\frac{a}{r}\right)^{n(k)+1} \frac{(n(k)+1)}{r} \sum_{m=0}^{k} (\overline{C}'_{n(k)),m} \cdot \cos m\lambda' + \overline{S}'_{n(k),m} \cdot \sin m\lambda') \cdot P_{n(k),m}(\cos\theta')$$

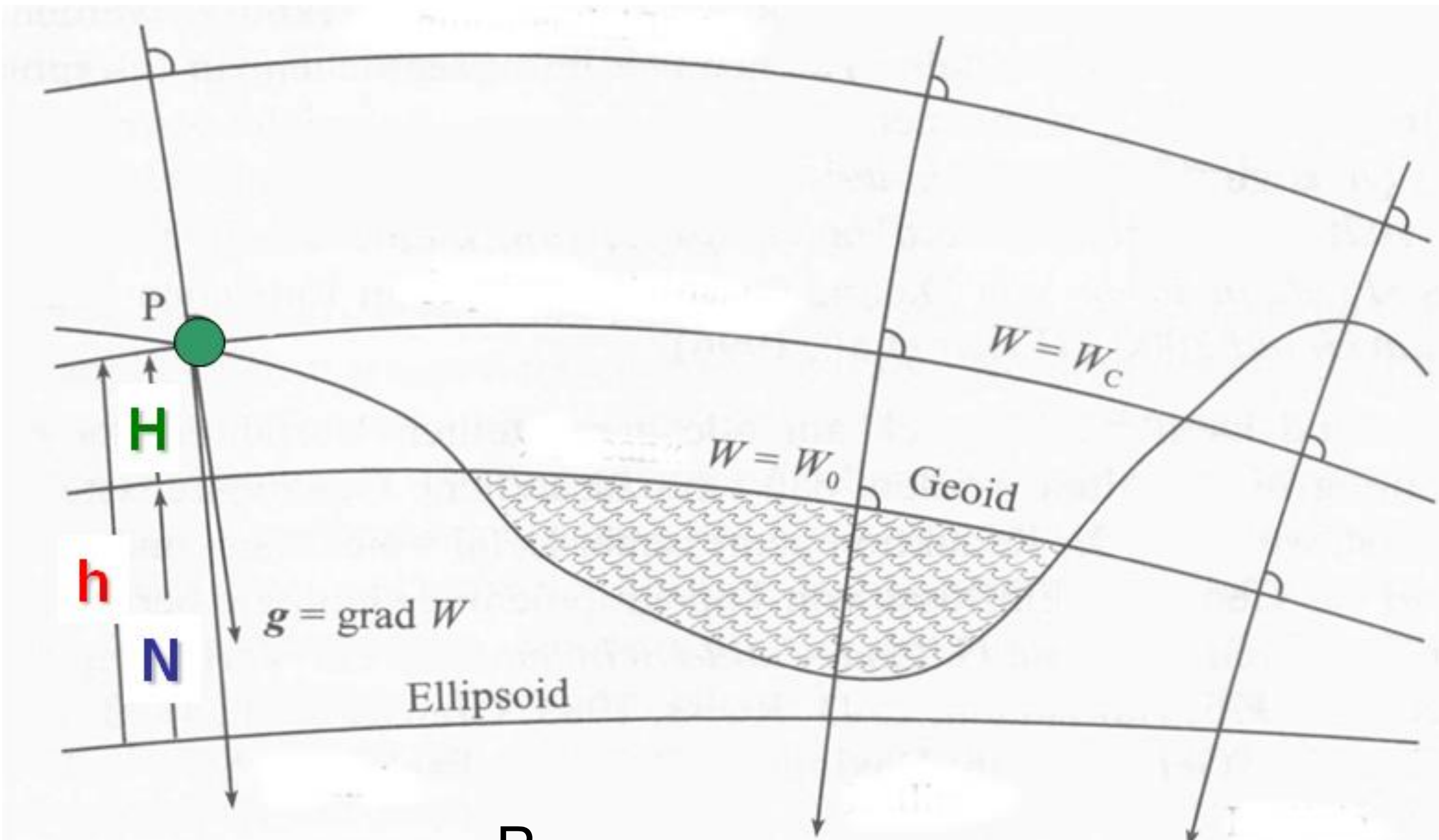
3. Vertical Deflections from Zenith Cameras **(T = (V+Z) – U)**

$$\xi = -\frac{\partial N_{QG}}{\partial B} \cdot \frac{\partial B}{\partial s_N} = -\frac{\partial B}{\partial s} \cdot \frac{\partial N_{QG}}{\partial B} = \frac{-1}{(M+h)} \cdot \frac{1}{\gamma_Q} \cdot \frac{\partial}{\partial B} T_p = \frac{-1}{\gamma_Q \cdot (M+h)} \cdot (\frac{\partial T}{\partial B})_P$$

$$\eta = -\frac{\partial L}{\partial s} \cdot \frac{\partial N_{QG}}{\partial L} = \frac{-1}{(N+h) \cdot \cos B} \cdot \frac{1}{\gamma_Q} \cdot \frac{\partial}{\partial L} T_P = \frac{-1}{\gamma_Q \cdot (N+h) \cdot \cos B} \cdot (\frac{\partial T}{\partial L})_P$$

Vertical Deflections Zenith-Cameras

4. Levelling: By Gepotential Numbers



$$\Delta C_{P_{j,i}} = \sum_{P_i}^{P_j} \overline{g}_{k,l} \cdot \Delta H_{k,l} = W_{P_i} - W_{P_j}$$

Sensor Network Geometric Space – 3D Parametrization in Integrated Approach

5. TPS

$$\mathbf{l}_{ij}^{LAV,i} = \mathbf{D}_i(\varphi_i,\lambda_i)\cdot\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix}_{ij}^{ECEF} + \begin{bmatrix}0\\ 0\\ -i\end{bmatrix}^{LAV,i} + \mathbf{D}_i(\varphi_i,\lambda_i)\cdot\mathbf{D}_j(\varphi_j,\lambda_j)^T\cdot\begin{bmatrix}0\\ 0\\ t\end{bmatrix}^{LAV,j}$$

$$\mathbf{l}_{ij}^{LAV,i} = \begin{bmatrix}\Delta u\\ \Delta v\\ \Delta w\end{bmatrix}_{ij}^{LAV,i}$$

TPS Total-station

$$s_{ij} = \sqrt{\Delta u^2 + \Delta v^2 + \Delta w^2}$$

3D-Slope Distance

$$r_{ij} = \arctan\left(\frac{\Delta v}{\Delta u}\right) - o_i$$

Direction

$$z_{ij} = \arctan\left(\frac{\sqrt{\Delta u^2 + \Delta v^2}}{\Delta w}\right) - \frac{s_{ij}}{2R}\cdot k$$

Zenith Angle

$$\mathbf{D}_e^{LAV} = \begin{bmatrix}-\cos\lambda\cdot\sin\varphi & -\sin\lambda\cdot\sin\varphi & \cos\varphi\\ -\sin\lambda & +\cos\lambda & 0\\ \cos\lambda\cdot\cos\varphi & \sin\lambda\cdot\cos\varphi & \sin\varphi\end{bmatrix}$$

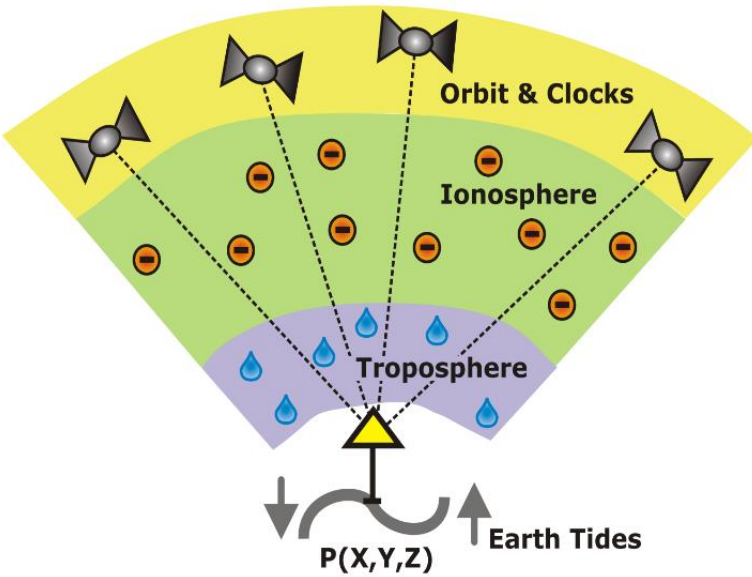
Geopotential-Modell W (C_{nm}; S_{nm})

$$\varphi = \arctan\frac{-W_z(C'_{nm}, S'_{nm})}{\sqrt{W_x^2)(C'_{nm}, S'_{nm}) + W_y^2(C'_{nm}, S'_{nm})}}$$

$$\lambda = \arctan\left(\frac{W_y(C'_{nm}, S'_{nm})}{W_x((C'_{nm}, S'_{nm})}\right)$$

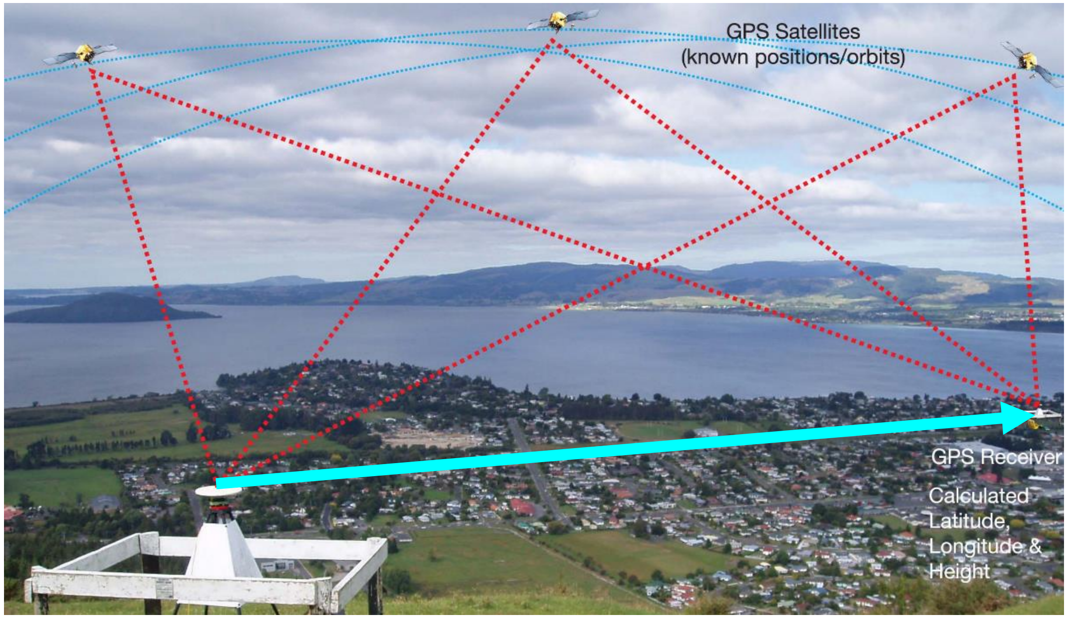
6. GNSS-Positions and Baselines

GNSS OPPP-Positions



$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

GNSS-Baselines



$$\mathbf{b} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \mathbf{x}_{Rover} - \mathbf{x}_{Base}$$

Conclusions

Quasi-Integrated and Integrated 3D Geomonitoring Approaches

- 3D necessary for sensor and parameter integration
- 2D/1D will has to remain, e.g. for 1D subsidence geomonitoring

Integrated 3D Geometry & Gravity Space Adjustment in Geomonitoring

- Feasible based on Regional Spherical Cap Harmonics Coefficients (Cnm',Snm')
- Integrates all relevant
    - Geometrical data
    - Physical data (regional geodynamics networks)

Quasi-Integrated and Integrated Geometry & Gravity Space

- Model Standards not defined yet
- Model validation for possible with Virtual GOCA software

- Integrated Geometry & Gravity Space Model - all sensors (RaD stuff)

- Quasi-Integrated 3D model implemented in GOCA-software – most of all sensors

- Observation Equation of Integrated and Quasi-Integrated Modelling can be mixed up


THANK
YOU
THANK